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Mathematically Gifted and Talented Learners: Theory and Practice

Authors:

Professor Valsa Koshy
Brunel University
Uxbridge UB83PH
United Kingdom
Valsa.koshy@brunel.ac.uk

Professor Paul Ernest
School of Education
Exeter University
Exeter EX! 2LU
P.Ernest@exeter.ac.uk

Ron Casey
Senior Research Fellow
Brunel University
Uxbridge UB83PH
United Kingdom
Ronald.casey@brunel.ac.uk

Nurturing Mathematically Gifted Learners: Theory and Practice

Valsa Koshy, Paul Ernest and Ron Casey

Brunel University, Exeter University, Brunel University

Abstract

There is growing recognition of the special needs of mathematically gifted learners. This paper reviews policy developments and current research and theory on giftedness in mathematics. It includes a discussion of the nature of mathematical ability as well as the factors that make up giftedness in mathematics. The paper is set in the context of current developments in Mathematics Education and Gifted Education in the UK and their implications for Science and Technology. It argues that early identification and appropriate provision for younger mathematically promising pupils capitalises on an intellectual resource which could provide future mathematicans as well as specialists in Science or Technology. Drawing on a Vygotskian framework, it is suggested that the mathematically gifted require appropriate cognitive challenges as well as attitudinally and motivationally enhancing experiences. In the second half of the paper we report on an initiative in which we worked with teachers to identify mathematically gifted pupils and to provide effective enrichment support for them, in a number of London Local Authorities. A number of significant issues are raised relating to the identification of mathematical talent, enrichment provision for students and teachers' professional development.

Keywords: Mathematical giftedness, enrichment, Zone of Proximal Development, mathematics learning, science, technology

1. Introduction

For the past three decades there has been growing recognition not only of individual differences between learners but that some learners have special educational needs and gifts. In Great Britain the Warnock Report [1] claimed that “at any one time about one child in six is likely to require some form of special educational provision.” This applies at least as much in mathematics as in other school subjects where a ‘seven year difference’ has been remarked in performance on certain mathematical tasks in the Cockcroft Report [2]:

> *[T]here is a ‘seven year difference’ in achieving an understanding of place value which is sufficient to write down the number which is 1 more than 6399.* By this we mean that whereas an ‘average’ child can perform this task at age 11 but not at age 10, there are some 14 year olds who cannot do it and some 7 year olds who can.

This illustrates graphically the dramatic divergence of understanding and performance in mathematics. Some primary school children will not achieve the level of understanding this particular task requires until half way through secondary schooling, and others will already have achieved it by the time they enter junior school.

However, a common view until the Cockcroft Report [2] was published was that mathematically gifted students were not an educational issue or ‘problem’, for through their talents they could take care of themselves. The authors of The Cockcroft Report felt the need to explicitly refute this view.

> The statement that able children can take care of themselves is misleading, it may be true that such children can take care of themselves better than the less able, but this does not mean that they should be entirely responsible for their own

> programming, they need guidance, encouragement and the right kind of opportunities and challenges to fulfil their promise.

At the same time, Anita Straker [3], who was later to direct the National Numeracy Strategy in the UK, published one of the first books devoted to mathematical giftedness in which she argued for the importance of attending to such students:

> Gifted pupils have a great deal to contribute to the future well-being of the society, provided their talents are developed to the full during their formal education. There is pressing need to develop the country's resources to the fullest extent, and one of our most precious resources is the ability and creativity of all children.

Progress in implementing strategies for educating gifted mathematicians in the UK had been slow, until the launch of the 'Gifted and Talented' initiative by the government, which has given a new impetus to this topic. Recorded commitment to gifted education can be seen in the 'Excellence in Schools' document [4] sending a strong message that all schools should seek to create an atmosphere in which to excel is not only acceptable, but desirable. This was followed by action and generous funding through the launch of the Excellence in Cities initiative [5]. Local Education Authorities are required to identify their mathematically gifted pupils and provide a distinct teaching and learning programme for them. So the issue of mathematically gifted students is no longer a neglected one in the UK.

The developments relating to provision for gifted mathematicians in UK schools coincide with other developments in Mathematics Education and the importance of effective teaching and learning mathematics in the context of Science and Technology. The need for reform in Mathematics Education was highlighted in Professor Adrian Smith's [6] assertion in his extensive enquiry into Post -14 Mathematice Education:

> ..it is widely recognised that mathematics occupies a rather special position. It is a major intellectual discipline in its own right, as well as providing the underpinning language for the rest of science and engineering and, increasingly, for other disciplines in the social and medical sciences. It underpins major sectors in modern business and industry , in particular , financial services and ICT.

Smith's report highlights the situation of a long decline in the numbers of young people continuing to study mathematics post -16. It draws attention to possible factors underlying this decline. Among those are the perceived poor quality of teaching and learning, the failure of the curriculum to excite interest and provide appropriate motivation and many young people's perception of mathematics as "boring and irrelevant". The shortage of specialist mathematics teachers and the continued fall in numbers of graduates in mathematical sciences were highlighted in 'More Maths Grads' (Higher Education Academy Subject Centre for Maths , Stats &OR [7]. Whilst the forementioned documents highlight legitimate concerns about Mathematics Education in Secondary schools and Higher Education, the William's Review [8] justifies the design of strategies and allocation of resources targeting primary school teachers and children. This review emphasises the need to enable learners in primary and Early Years settings to acquire an understanding and appreciation of mathematics and its importance in their lives. The Williams Review makes strong recommmenadations for Continuing Professional Development for teachers to enhance the quality of mathematics teaching.

An intervention programme for mathematically promising pupils in Upper Primary School (Year 5 – pupils, aged 10) was commissioned by the UK government, which is described in Section 3 of this paper. The educational challenge was to generate interest in the potential of mathematics to enhance the life-worlds of children already enjoying the benefits of text messaging, the internet and iPods unaware of the contribution of mathematics to their creation. The justification of intervention for mathematically gifted children in the Primary Phase is that their life-course could subsequently contain decades of creative dedication to Mathematics, Science and Technology. It is noteworthy that Fermat's Theorem was met by a 10 year old who decades later provided its proof.

2. Mathematical ability, giftedness and talent

An ability is the quality of being able to do something; a natural or acquired skill or talent. Thus mathematical ability is the quality of being able to do mathematics that is being able to perform mathematical tasks and to utilise mathematical knowledge effectively. This could be in selected areas of mathematics or more widely. For school students, mathematical ability is normally manifested in accomplishing tasks related to the mathematics curriculum. However, there is also a further dimension to mathematical ability, namely a potential or future-oriented skill; the capacity to learn and master new mathematical ideas and skills, as well as to solve novel and non-routine problems. Because of this dimension, mathematical ability is not observable. It can only be inferred from observed performances and therefore remains elusive and any diagnosis of its extent must remain tentative and conjectural. As a broader and more sustained pattern of responses is observed to a range of tasks and circumstances a better picture of a student's mathematical ability emerges. But no final diagnosis can ever be achieved because there is always the possibility for significant experiences to trigger an unexpected shift in student performance levels, both upwards and downwards.

Ability in other subjects may or may not be linked to mathematical ability. Vernon [9] claims that verbal ability tends to correlate negatively with mathematical ability. In contrast, Hadamard [10] suggested that it was doubtful that mathematical aptitude existed as separate from aptitude generally. Wrigley [11] found that 70% of mathematical attainment could be identified with a factor which he termed 'general ability'. General ability is often identified with intelligence. Likewise, Guilford [12] found 50% to 80% correlation between general measures of ability and mathematical attainment. Such views imply that mathematical ability is something constant and enduring over a person's life strongly correlated with measured intelligence (IQ).

A key way in which labelling works is that student attitudes and beliefs about mathematics, including their own perceptions of their own ability in the subject, become internalized and reinforced by their experiences (and behaviours). The outcome can be self-fulfilling and self-reinforcing cycles. These can be either a negative, vicious cycle termed the 'failure cycle', or a positive, virtuous cycle, termed the 'success cycle' by Ernest [13]. These are further discussed below concerning the impact of affective factors on performance.

The psychological critique is based on the view that using overall measures of general intelligence or mathematical ability is problematic. Hann and Havinghurst [14] report that "another school of thought prefers to think of intelligence as consisting of as many as 6 or 7 different abilities which are interrelated." More recently, Howard Gardner [15,16] has developed a theory of multiple intelligences. He distinguishes intelligence and abilities and asserts that it is "reasonable to think of a particular area of ability as forming a distinct intelligence only when there is supporting evidence from a number of different sources." He claims that we have several different and independent skills/abilities ('multiple intelligences'), including logical-mathematical, linguistic, visual-spatial, bodily-kinesthetic, musical, interpersonal, intrapersonal, and naturalistic, and that consequently intelligence is not a unity. His theory of eight or so types of intelligence is based on neuropsychological analysis of human abilities. One of Gardner's specific intelligences is referred to as 'Logical Mathematical Intelligence'. Within this intelligence Gardner perceives a progression from object manipulation to pure abstraction, which takes into account different elements of mathematical ability, but is also reminiscent of Piaget's theory. Research by Michael Howe studying *idiot savants* has given a new source of evidence to reinforce theories of multiple intelligences. His study has shown that "separate mental abilities can exist in relative isolation, to a large extent independently of a person's measured intelligence" [17].

At the forefront of any discussion of mathematical ability is the argument about whether or not it is innate, whether it is due to nature or nurture. Although still controversial, in our view this is settled: both are important. For example, Krutetskii [18] has written that "mathematical abilities are not innate, but are properties acquired in life". He also writes "some persons have inborn characteristics in the structure and functional features of their brains which are extremely favourable to the development of mathematical abilities." This suggests that while actual mathematical abilities are not innate, some children will become more able than others because of inborn characteristics, which are later developed and actualized into mathematical ability through subsequent development and experiences. According to constructivist theories of learning [19] these experiences form the basis on which learners construct knowledge and understanding in their own personal and sometimes idiosyncratic ways.

Acknowledging the central role of experience for the development of mathematical ability, not least for persons with mathematical gifts and talent, underscores the importance of appropriate stimulation, guidance and teaching. This includes both the stimulation of interest, for example, through puzzles, patterns and other intriguing phenomena in mathematics, and exposure to the concepts and ideas of mathematics, at an appropriate level of cognitive challenge. Such teaching and stimulation ideally should take place in school mathematics classes, at home, and possibly in other organised activities such as mathematics masterclasses or enrichment activities, as well. In each of these cases there is an essential role for more knowledgeable others.

Social constructivist theories of learning, drawing on Vygotsky [20], place social interaction such as with the teacher as central to the development and extension of a learner's capabilities. One of Vygotsky's central ideas is that of the zone of proximal development (ZPD). He defines this as "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers.". In other words, the ZPD lies beyond the area of problems that a child can do unaided (which we shall name the zone of accomplishment), and includes the tasks that the child can only do with the help of a teacher, peer or parent. Through experience and development the child masters some of the abilities and tasks involved in the ZPD and they become part of the zone of accomplishment. At the same time some of the tasks that were unattainable, even with the help of others, now become accessible in the ZPD. Thus teaching plays a key role in bringing tasks and capabilities within reach. Often it provides 'scaffolding'; models for imitation in problem solving, as well as guidelines, instructions, work routines and so on . As an instructional principle, scaffolding requires a gradual withdrawal of support within the ZPD as the child's knowledge and confidence increase.

2.1 Attributes of the mathematically gifted and talented

Krutetskii [18] carried out an extensive investigation into the psychology of mathematical abilities in schoolchildren in Soviet Russia. He developed a checklist of the key elements of mathematical thinking observed in the mathematically gifted and talented:

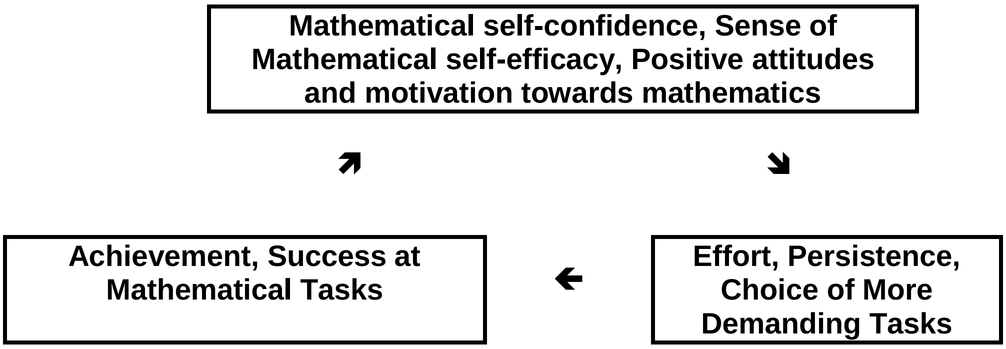
- An ability to formalize mathematical material, to isolate form from content, to abstract oneself from concrete numerical relationships and spatial forms, and to operate with formal structure - with structures of relationships and connections.

- An ability to generalize mathematical material, to detect what is of chief importance, abstracting oneself from the irrelevant, and to see what is common in what is externally different.
- An ability to operate with numerals and other symbols.
- An ability for ‘sequential, properly segmented logical reasoning’, which is related to the need for proof, substantiation, and deductions. …
- An ability to shorten the reasoning process, to think in curtailed structures.
- An ability to reverse a mental process (to transfer from a direct to a reverse train of thought).
- Flexibility of thought - an ability to switch from one mental operation to another; freedom from the binding influence of the commonplace and the hackneyed. This characteristic of thinking is important for the creative work of a mathematician.
- A mathematical memory. It can be assumed that its characteristics also arise from the specific features of the mathematical sciences, that this is a memory for generalizations, formalized structures, and logical schemes.
- An ability for spatial concepts, which is directly related to the presence of a branch of mathematics such as geometry (especially the geometry of space).

2.2 Affect and the mathematically gifted and talented

There is a further area, mentioned above, whose importance for the mathematically gifted is often underestimated. This is the central role of the affective domain, including motivation and attitudes. This impacts on both of the other two dimensions in the model of giftedness of Ridge and Renzuli [21], namely cognitive abilities and creativity. Aiken [22] conducted research on creativity in mathematical activity and concluded that affective variables make a central contribution to mathematical ability and its creative applications. The impact of affect on cognitive performance and abilities is more complex. One simple model of this interaction is the ‘success cycle’ [13].

Figure 1: The Success Cycle



The success cycle has three components:

1. Positive affect including attitudes and motivation towards mathematics,
2. Effort, persistence, and engagement with cognitively demanding tasks,
3. Achievement and success at mathematical tasks

The success cycle has no real beginning because these three components are linked cyclically, and each impacts positively on its successor. Taking 1 as a starting point we can say that students who have positive attitudes and beliefs about mathematics typically have high mathematical self-confidence and a good sense of mathematical self-efficacy. They enjoy doing mathematics and experience pleasure in the challenges it presents. This positive motivation leads to increased effort, persistence, more time on task, and the choice of cognitively more demanding tasks in mathematics. Given appropriate opportunities the increased effort and work will give rise to continued success at mathematical tasks and overall achievement in mathematics. This sustains and further enhances positive attitudes, completing the success cycle and giving it more momentum. This gives rise to a virtuous, upward spiral. Figure 1 illustrates the cycle diagrammatically.

3. ACTUALIZING MATHEMATICAL GIFTEDNESS – A TWO YEAR STUDY

On the basis of this review of research, a pilot project was launched to actualize mathematical giftedness in a sample of promising students at the end of Key Stage 2. This was a 2-year study carried out by a research team at Brunel University, and it highlighted some significant issues relating to both the identification of and provision for mathematically gifted students [23, 24]. The study, carried out with pupils aged 10-11, within inner-city schools, was funded by the British government as part of the Excellence in Cities initiative [5]. A number of components, which may enhance provision for mathematically gifted pupils, were identified by the team and should be of interest to a wider audience.

The main aim of the project was to identify possible factors which contribute to the effective identification of and provision for mathematical promising pupils. In order to achieve this aim, a Mathematics Enrichment Project (MEP) was set up with the following two interrelated objectives:

- to support Local Education Authorities to set up enrichment programmes for groups of mathematically gifted children
- to provide a professional development programme for teachers to support them in delivering a distinct teaching programme for the participating children.

Twelve Local Education Authorities (LEAs) within Inner London were invited to take part in the project and 11 of the 12 decided to participate. As part of the project, each LEA was also invited to set up a mathematics enrichment group of 25- 30 mathematically gifted children, nominated by their teachers, to meet at a local venue. This meant that a total of up to 330 Year 5 (9- 10 years old) children would be able to participate in the programme. The intention was to provide practitioners with an opportunity to focus on aspects of mathematical promise and explore ways of fulfilling that promise. The choice of the age group was decided by the LEAs in discussion with the research team, on the basis that this particular age group would have spent at least 7 years in school and would be taking the end of Key Stage 2 national tests in the following year before transferring to secondary education. Each group of children was to be taught by teachers who wished to develop their own expertise in meeting the needs of gifted children, by attending a professional development programme at the University. Four full days of attendance per academic year was required of the teachers (33 attended in total). The enrichment classes for the students were organized on Saturday mornings for 2 hours and a total of 20 sessions, over two years, were taught to each group of pupils.

3.1 Methodological issues

From the outset, it was emphasized to all the participants that the purpose of the study was to highlight issues relating to meeting the educational needs of a particular group of children who were referred to as *gifted* in mathematics. An action research model was used in which practitioners followed a cycle of planning, teaching, collecting information on the intervention, evaluating its effectiveness and reflecting on the outcomes. Teachers kept a field diary in which they made notes of significant incidents. The 11 Local Education Authorities that participated in the project were all located within the Inner London area. The LEAs included a range of schools of different sizes and socio- economic mix, as judged from the number of free school meals. Half the authorities had a significant number of pupils for whom English was an additional language. Six of the LEAs were amongst the 20 lowest achieving authorities (out of 150) in England and Wales in the national tests for mathematics.

Data sources included:

- questionnaires completed by Local Education Authority mathematics advisers and Year 5 class teachers who were asked to comment on their experiences of how they selected the children for the programme.
- semi-structured interviews with 12 Year 5 teachers.
- field dairies, kept by the teachers who attended the professional development, on aspects of their training and their experiences at the enrichment classes.
- a detailed semi-structured evaluation of all aspects of the professional development programme, completed by the participating teachers at the end of the professional development programme. They were asked to list any aspects they may have found particularly challenging or useful to their professional development within the context of selecting and teaching the children.
- observation notes made by the authors of this paper and an external evaluator who observed the enrichment classes six times a year. The notes were subsequently shared with the teachers.
- Pre- and post- project questionnaires, completed by the children in September and July – at the beginning and end of the teaching sessions. They were asked to comment on their perceptions of what mathematics was about, how they saw their school mathematics lessons and how they felt about being selected for the enrichment project.

3.2 Theoretical basis of the project

The model used for the intervention programme adopted a (social) constructivist view of learning which accounts for the ‘individual and idiosyncratic constructions of meaning’ [19]. This view encourages active participation in the lessons and is based on the belief that the learner constructs his or her own learning. In the context of this project, teachers were encouraged to familiarize themselves with the concept of constructivism and were asked to view themselves as *constructivist* learners. They were also encouraged to use discussions and on-going formative assessment of students' performance to ascertain the level of their cognitive involvement and any sources of difficulty. Learning diaries, which recorded significant events, thoughts and reflections during sessions, were kept by both teachers and pupils. Two principles for designing learning tasks were shared with the teachers. First, the level of challenge and cognitive demand built into the tasks needs to ensure that children's real potential, within Vygotsky's Zone of Proximal Development, could be recognised and acted upon. Second, the important role played by adults and peer groups for the effective construction of knowledge was stressed during training sessions. Learning from the more knowledgeable adults was an important aspect of the programme organization. Cognitive demands of the learning tasks were guided by incorporating higher levels of thinking – analysis, synthesis and evaluation - offered by Bloom's Taxonomy [25] and taking note of Resnick's [26] recommendations that higher order thinking involves providing children opportunities for multiple solutions and freedom of enquiry without pre-determined outcomes .

Teaching styles on both the professional development programme and children's taught sessions adopted an interactive format. The importance of discussion and raising questions was acknowledged as an integral part of effective learning. Reynolds and Muijs's cautionary words [27] that whilst whole-class interactive teaching may be particularly useful in maximizing achievement in standard tests and basic skills, this approach may not be sufficient for teaching higher order mathematical problem-solving or mathematical thinking skills, were taken into account when considering teaching strategies.

3.3 Findings

Data gathered over the two-year period highlighted a number of issues relating to the actualization of mathematical talent; the significant pointers have been published [23,24] and disseminated at conferences in the UK. Some issues which are considered significant, in the context of this paper, are raised and discussed in the next section. These are based on the analysis of data from questionnaires and the content of interviews, field notes and teachers' evaluations, which were analyzed for emerging 'categories' and themes, using guidance from Strauss and Corbin [28]. Four themes are presented:

- Issues relating to the identification of mathematical talent
- Issues relating to provision
- Issues relating to teachers' professional development
- Children's responses to the enrichment project .

3.3.1 Issues relating to the identification of mathematical talent

Teachers found the process of selecting a group of gifted mathematicians to be one of the most challenging aspects of the project (72% of a total of 67 who completed questionnaires referred to this). A sub sample of 15 teachers who were interviewed raised two types of difficulties.

The first was the difficulty of observing the attributes of mathematically gifted pupils within the recommended structure of mathematics lessons (a three-part mathematics lesson of 50 minutes length – as suggested by the National Numeracy Strategy [29]. Liz, a teacher with 12 years of experience of teaching in primary schools trying to use the criteria provided by the research team which was based on Krutetskii's work [18] for identifying gifted students for the enrichment programme, described what many of the teachers felt:

I feel that the lessons are quite rushed as I want to keep to the time guidelines of 10 minutes mental starters, 30 minutes for the main activity and the last 10 minutes for a plenary to discuss the learning outcomes of the lesson. The pace of the lessons and the whole class teaching strategies I employ make it difficult to observe the attributes I am looking for. I am thinking that it would be worth revisiting my lesson plans and introduce activities which *provide more opportunities for children to demonstrate their true potential ability. I feel I am always rushing or teaching what may appear in tests these children have to take next year.*

A second difficulty raised in the context of identifying mathematical talent was that when teachers identified promising mathematical ability in the classrooms, many of their ‘best’ students could only be described as *‘average’* within the UK National Curriculum assessment. This aspect made teachers feel uncomfortable about labelling them *gifted* and recommending them for a programme designed for mathematically gifted pupils. Just over half the number of teachers believed that their students’ achievement in tests could have been affected by their background of social deprivation, disadvantage, low expectations and the lack of adult support.

Data from interviews also suggested that the problems with identification may have also been contributed to by students’ low level of performance in formal tests. During the enrichment classes, teachers found many students lacking in factual knowledge and skills which made it difficult for them to undertake investigational tasks which required speed and fluency in arithmetic procedures. Teachers who taught the enrichment groups recorded in their field-notes that quite often they had to implement ‘catching up’ strategies for many of the participating children and that they had not anticipated the need for this. Significantly, some teachers also raised an important point that most of these children *caught up* very fast with basic skills and were able to tackle complex tasks after a short period of attendance. Reasons for the above observations can only be speculated on, but it is likely that the children who were nominated for the programme did possess potential mathematical ability; but external factors such as teacher shortages in inner-cities, discipline problems and lack of family support created a mismatch between the children’s level of achievement and their true potential.

The findings of this study highlight the need for careful consideration of the whole notion of identification of mathematical ability. Inclusion issues in the identification of mathematical talent have also posed a major challenge in the United States. It is interesting to note that the National Council of Teachers of Mathematics in the US appointed a Task Force, in 1995, on mathematically promising students and asked the group to rethink the traditional definition of mathematically gifted pupils to broaden it to the more inclusive idea of mathematically promising students. The task force redefined the category of students to go beyond the traditional membership of the gifted category as the top three to five percent of students based on some standardised mathematics test [30]. Teachers who participated in this study also found the term 'mathematically promising', used by the American Task Force, more acceptable . They were also in agreement with the view of the US Task Force that an outdated definition of giftedness, based on test results alone, frequently unnecessarily restricts access to interesting, challenging mathematics to a small portion of the population. The Task Force redefined mathematically promising students [30] as a function of variables that ought to be maximized. In their report, mathematical promise is describes as a function of:

- *ability*
- *motivation*
- *belief*
- *experience and opportunity*

and it points out that these variables were ones that could and should be developed in all students if we are going to maximize the number of students with mathematical talent. Sheffield [31] provides further support for this view by citing recent brain functioning research that documents changes in the brain due to experiences. She cites Clarke's [32] assertion to support the argument:

No child is born gifted – only with the potential for giftedness. Although all children have amazing potential , only those who are fortunate enough to have opportunities to develop their uniqueness in an environment that responds to their particular patterns and needs will be able to actualize their abilities to high levels. She reminds us that the brain grows and develops as it responds to challenging problems and mathematics is a perfect venue for this development.

3.3.2 Issues relating to provision for mathematically promising students

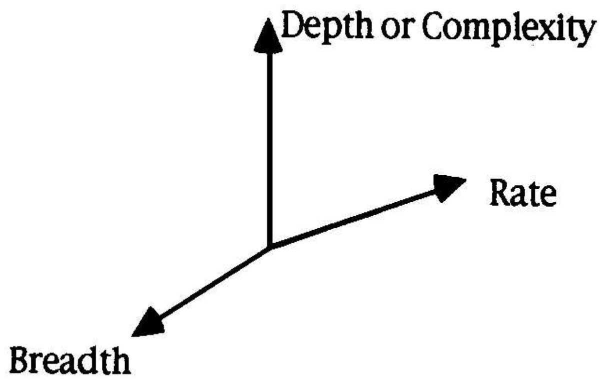
At the start of the professional development programme, the participants pointed out that none of them (33 teachers) had received any special training or support in addressing aspects of teaching able mathematicians. Two strategies employed by them consisted of either using teaching materials and activities from higher levels of the National Numeracy Strategy [29] or preparing additional work based on the same content for students who '*finished'* their work before their peers. As the strategy for provision adopted for the MEP was one of enrichment, the on-going debate on whether to use *acceleration* or *enrichment* as the preferred strategy for teaching mathematically promising students dominated much of the discussions within the professional development programme. Evidence from teaching sessions and pupils' work highlighted the complexity of selecting one or the other strategy. Two significant issues emerged. First, pupils responded , in most cases, to open-ended investigational tasks with greater motivation than they did to mathematical exercises and tasks selected from mathematics text books designed for older age groups. Second, if motivated by an investigational task, children tended to seek knowledge and skills required from higher levels on the mathematics curriculum without being prompted to do so. In the light of this experience the project teachers felt more at ease with the enrichment strategy.

The *acceleration – enrichment* debate needs to be considered within the context of the support from the government for fast- tracking and early entry to GCSE examinations [4,5] and the criticism levelled at the acceleration model raised by a group of mathematics educators [33]. They describe the acceleration strategy as having *'serious disadvantages for pupils' long term development'* and needing to be '*handled with great caution'.* Enrichment for *'added breadth'* is recommended as a way forward which involves providing:

> *Extension work which enriches the official curriculum by requiring deeper understanding of standard material (for example, by insisting on a higher level of fluency in working with fractions, ratio, algebra or in problem solving.*

Other educationists [34] have also previously expressed concern with the acceleration strategy by asserting that in this model *pupils do not learn more about mathematics. What they do is merely learn the same mathematics sooner'.* This, he says, does not seem to fulfil the needs of the more able who deserve something better. In this context, it is interesting to note that Sheffield [31] the Chairperson of the US Task Force looking into provision for mathematically promising students, urges us to drop the *acceleration versus enrichment* debate and suggests that what we need to take into account is the depth of mathematics being learned. She reminds us of the criticism raised by Schmidt *et al* [35] that the US mathematics curriculum is *'a mile wide and an inch deep"*; that is, US students often learn a little bit about everything (and repeat that little bit for years) without exploring the rich depths of mathematics.

Figure 2: Sheffield's 3-dimensional model of curriculum factors for the mathematically promising



Sheffield's 3-dimensional model, in Figure 2, and her assertion that educators of mathematically promising pupils should look not only at changing the rate or the number of mathematical offerings but also at changing the depth or complexity of the mathematical investigations, was welcomed by the course participants. In the context of her involvement in other projects for the gifted, Koshy [36] found that many of the mathematical tasks offered to very able mathematicians included both enrichment and acceleration in the sense that when pupils were motivated to carry out a complex and in-depth investigation, they often sought new and more advanced knowledge.

3.3.3 Issues relating to teachers' Professional Development

Teachers who attended the professional development part of the project, which was a pre-requisite for teaching the enrichment group, felt it was '*timely'* and a '*much needed'* initiative considering the speed in which they were being asked, by the government, to implement new initiatives within both mathematics and gifted education. Schools were being set targets to increase the number of 11- year old children obtaining Level 5s in the UK National

Curriculum (average is Level 4) tests. Teachers were also being asked to include more problem - solving and reasoning within their mathematics teaching targeted at able mathematicians. In the light of these requirements, imposed centrally, it was felt that professional development support in aspects of realizing children's mathematical potential was either 'patchy' or 'non – existent'.

Written evaluations from the course participants also highlighted the need for professional development in aspects of teaching gifted pupils, as illustrated by comments such as:

> *All teachers must attend a training programme for teaching able pupils. School inspection teams are looking for evidence of good differentiation. Where does it come from? Where are the exemplified models. This course, considering special provision for gifted mathematicians, has enriched my own mathematics teaching and learning . By offering this kind of subject-based professional development to all teachers ,just think of the hundreds of children who will benefit, not just the 30 children who attend the classes on Saturday.*

The opportunity to work through mathematics investigations and experience the processes which teachers are expected to build into their classroom teaching was listed as the *most useful part of the professional development part of the project.* Teachers felt more *'confident'* and more *'positive'* to take on the challenge of teaching promising mathematicians and claimed that their personal learning of mathematics had led them to reconsider their teaching styles . It seemed that the course content and style had helped to ease the general anxiety of primary school teachers about their lack of subject knowledge. Teachers' comments included:

> *Understanding algebra, those letters and formulas, after all these years, was very exciting. The teaching style of the course programme offered a great model for how*

we should teach in the classroom. Making decisions without fear, playing around with mathematical ideas and feeling that it is OK to get stuck are worthy of being established in our own classrooms.

I have always felt that my subject knowledge was not good enough to teach very able kids. Working through the resources enhanced my subject knowledge in a very short time, but more importantly, I am now convinced that you don't need to know everything before asking children to do an investigation or problem solving activity. I feel confident to say "Let us work this out together , I have not cracked it either…"

The above comments bear testimony to the claim by HMI [37] that efforts in meeting the needs of the very able can often help to raise both teacher expectations and pupil achievement. Professional development of teachers has been highlighted as one of the factors contributing to the effective teaching of numeracy by Askew *et al*[38] and it may be that in the context of teaching mathematics to gifted mathematicians it is of particular importance, considering the general perception of teachers' lack of subject knowledge and the deficit model of the teacher highlighted by Jeffery *et al* [39].

It is interesting to note that although many distance learning, web-based materials and teaching strategies are made available, by the UK government, offering support for the practising teacher, teachers stressed the importance of 'face to face' training as they felt *teaching mathematics to gifted children to be a particularly complex and challenging activity.* In the light of this observation it was not surprising to note that only a very small number of participating teachers (9%) had been aware of the resources offered by the UK Department of Education and the Qualification and Curriculum Authority; even those who had heard about them had not used them. This issue is of some significance for policy makers.

3.3.4 Children's responses to the enrichment project

A noticeable change in children's perception of mathematics was evident. For example, in response to the question *what is mathematics?* in the pre-enrichment project questionnaire, 84% of the children described mathematics in terms of '*sums' 'learning all about numbers', 'learning to calculate', 'fractions and them stuff'.* The post-project questionnaire showed some changes in their perception . More than half the children who had previously described mathematics in terms of working with number or doing sums changed their perception and described the subject as involving '*solving problems', 'looking at information on charts and writing reports about it'*. They also included *'working with shapes'* and *'measuring'* in their definitions. In response to the question on what they *'felt about school mathematics'*, 74% of pupils had described mathematics lessons to be '*boring, repetitive'* and '*too easy'.* Some pupils commented that they found the work in the dedicated Numeracy Hour too easy, especially the *mental starters* and *plenary sessions* as they were excluded from answering any questions *even 'after hours of putting our hands up'.* In the post-project questionnaires children described the sessions as '*very enjoyable , but very hard',' making you think'* and '*teaching you to do whizzy things like adding all the numbers from 1 to 10000 within 3 minutes'. Having enough time to think* and teachers' acceptance of *mistakes in a positive way* were also listed as good features of the project.

Interviews with students and teachers showed that children had developed a sense of achievement during the project. Most of the participating children (94%) were proud to have been selected for the project. Based on the feedback received from a number of their class teachers, most of whom had not been involved in teaching the enrichment sessions, attendance at the Saturday programme had an impact on their class work in terms of both increased motivation and achievement. As no structured pre-and post-tests of achievement were used within the study, it is not possible to make any claims about any enhanced achievement in terms of test scores. However, an analysis of children's work collected during the enrichment sessions showed an improvement in the quality of their responses. There was evidence of more systematic work, use of tables and children making conjectures and extending their thinking by posing their own questions. Although the participating teachers attributed the changes in student's perceptions of mathematics and the enhanced quality of their work to being taught by teachers who had developed their own understanding and expertise, they also acknowledged that such claims can only be tentative in the absence of a control group.

4. Concluding remarks

The limitations of using a small sample which was also an opportunity sample [40], makes it difficult to make generalizations. An analysis of the data gathered from multiple sources for a period of 2 years has highlighted a number of factors which have implications for policy makers and practitioners in the actualization of students' mathematical promise.
The findings of the project reported in this paper suggest the following:

Early identification of mathematical and creative ability is important; this is particularly vital in the context of inner-city students who participated in the MEP project. It is a misconception – at all levels of an education system – to view lack of basic skills as indicative of limited intelligence. It is highly likely that very promising young students, like those who participated in our project, would have failed to achieve high grades in public examinations to secure places in Universities. The endeavours of the UK central government to encourage Widening Particaption, thereby enabling children from disadvantaged backgrounds to benefit from University education, needs to take account of the submerged talent which needs early detection and subsequent support. However, identification of mathematically gifted pupils is not a simple process. It cannot be assumed that teachers will automatically know how to select their best mathematicians. The concept of giftedness is complex and teachers need much more support through professional development programmes in order to develop an understanding of issues. Test results are only part indictors of high ability; teachers need to employ a range of methods to identify mathematically promising children. Close observation of gifted behaviours is a useful way forward as was shown during this project.

One of the significant findings that emerged, throughout the MEP project, is the importance of providing adequate professional development support in empowering teachers to enhance and enrich their own mathematical knowledge and methodological expertise in teaching mathematically promising children. This empowerment may only be achieved through teachers deepening their own understanding of the subject and its methodology, the development of a variety of teaching approaches and styles and being given opportunities to reflect on their own learning and sharing perspectives with colleagues. This observation is consistent with the recommendation by Ofsted [41] for subject-specific professional development of teachers as a means of achieving more effective provision for gifted pupils although this has not been addressed sufficiently. It is interesting to note that many of the issues that emerged during the MEP project are similar to what have been raised in the two major reviews on Mathematics Education in the UK [6 , 8]. For example, the need for more Professional Development programmes for teachers to enhance their mathematical subject and pedgagogial expertise and the importance of introducing strategies for children to be better motivated and have more in-depth knowledge of mathematics are among the recommendations made in the two reviews. The comment of Alison, one of the participants, is representative of the views of many:

> *For many primary teachers, like me with very little knowledge of mathematics, it is difficult to feel that we can cope with children who know more maths and are faster than us. I did not honestly know what kind of questions to ask or how to suggest extensions. Working with colleagues who had the same problem helped me to challenge my own perception of mathematics and how to teach it. In a way, what we have come to know and learn is how to teach mathematics effectively to all children.*

This will benefit all children in my class , not just a few. I feel that the course has enabled me to take risks within a supportive environment and helped me to grow in confidence, which is exactly what I would do with my children.

It is possible, in some cases that teachers' lack of mathematical subject expertise and confidence could transfer to their pupils who would displays lack of similar attributes. This could create an insurmountable obstacle to progress at seconday level and contribute to the dwindling supply of undergradtes.

A fundamental justification for early identification of mathematical talent and making adequate provision can be encapsulated in the following proposition: early care of the fertile soil of young minds will yield abundant future harvests of graduates in Science and Technology displaying mathematical proficiency. Students' responses, observed during the MEP project, showed that motivation may be regarded as correlated with relevance. Students of all ages need to feel that the curriculum has relevance to their lives and their society. Experiences in coding – such as Caeser codes, leading to exposure to simple examples of permutations - resulted in enhanced motivation. Questions such as why mobile numbers, passwords and PIN numbers require string lengths of characters could awaken dormant talent and inquisitiveness about similar facets of their life-world. The awareness that interesting questions require natural lanuguge need to be supplemented by the symbolism of mathematics should make future studies of modelling and equations palatable.

The education of young gifted mathematicians in the UK is at a critical crossroad. After many years of neglect there is now a real opportunity to develop systems which can benefit our promising mathematicians. Many unresolved issues - such as the relative merits of pull-out groups for gifted mathematicians, the relationship between the role of profession development of teachers and students' attitudes and motivation and an exploration of the best effective teaching methodologies - still remain and need further research.

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