

1 **Investigating Crack Initiation and Propagation of**
2 **Concrete in Restrained Shrinkage Circular/Elliptical**
3 **Ring Test**

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18 **Abstract**

19 The restrained ring test, which is recommended by AASHTO and ASTM, has been used for assessing
20 the potential of early-age cracking of concrete and other cement-based materials. Recently, a novel
21 elliptical ring test method has been proposed to replace the circular ring test method for the purpose of
22 shortening ring test duration and observing crack initiation and propagation more conveniently. In order
23 to explore the mechanism of this novel test method, a numerical model is developed to analyze crack
24 initiation and propagation process in restrained concrete rings, in which the effect of concrete shrinkage
25 is simulated by a fictitious temperature drop applied on concrete causing the same strain as that induced
26 by shrinkage. First, an elastic analysis is conducted to obtain the circumferential stress contour of a
27 concrete ring subject to restrained shrinkage. Combined with the fictitious crack model, a fracture
28 mechanics method is introduced to determine crack initiation and propagation, in which crack resistance
29 caused by cohesive force acting on fracture process zone is considered. Finite element analysis is carried
30 out to simulate the evolution of stress intensity factor (SIF) in restrained concrete rings subject to
31 circumferential drying. Cracking age and position of a series of circular/elliptical concrete rings are
32 obtained from numerical analyses which agree reasonably well with experimental results. It is found that
33 the sudden drop of steel strain observed in the restrained ring test represents the onset of unstable crack
34 propagation rather than crack initiation. The results given by the AASHTO/ASTM restrained ring test
35 actually reflects the response of a concrete ring as a structure to external stimulation, in this case
36 restrained concrete shrinkage.

37

38 **Keywords** *concrete; early-age cracking; restrained shrinkage; fracture mechanics;*
39 *crack propagation process; elliptical ring test.*

40 **Abbreviations**

- a : crack length
- A : exposed surface area of a concrete element
- d : steel ring thickness
- E : elastic modulus of concrete
- f_c : uniaxial compressive strength
- f_t : splitting tensile strength of concrete
- G_F : fracture energy
- K_I^s : stress intensity factor caused by applied load
- K_I^σ : stress intensity factor caused by cohesive force
- R_0 : inner diameter of circular concrete ring
- R_1 : major semi-axes of inner circumference of elliptical concrete ring
- R_2 : minor semi-axes of inner circumference of elliptical concrete ring
- t : age of concrete
- V : volume of a concrete element
- w : crack opening displacement
- w_0 : stress-free crack opening displacement
- w_s : displacement corresponding to the break point in bilinear σ - w relationship
- Δa : crack growth length
- ΔP : load increment
- σ : cohesive stress
- σ_s : stress corresponding to the break point in bilinear σ - w relationship

42 **Introduction**

43

44 Autogenous, drying or thermal shrinkage often occur in concrete elements or
45 structures, resulting in volume changes of concrete. If volume changes of concrete are
46 restrained, tensile stress may be developed in concrete [1, 2]. Since concrete's low
47 tensile strength provides little resistance to tensile stress, cracking may occur when the
48 developed tensile stress in concrete exceeds its tensile strength. These restrained
49 shrinkage cracks are often observed in concrete elements and structures with high
50 surface area-to-volume ratio such as industrial floors, concrete pavements and bridge
51 decks [3]. Cracking in concrete may reduce load carrying capacity, shorten the service
52 life of concrete structures, and accelerate deterioration resulting in increased
53 maintenance costs. Therefore, it is significant to assess cracking potential of concrete
54 mixtures under restrained shrinkage in service.

55 The restrained circular ring test has been widely used for assessing cracking
56 tendency of concrete and other cement-based materials due to its simplicity and
57 versatility [1, 4-10]. As a standard test method for assessing cracking potential of
58 concrete mixtures under restrained condition, the circular ring test has been
59 recommended by AASHTO (i.e. AASHTO T334-08: *Standard Method of Test for*
60 *Estimating the Cracking Tendency of Concrete*) and ASTM (i.e. ASTM
61 C1581/C1581M-09a: *Standard Test Method for Determining Age at Cracking and*
62 *Induced Tensile Stress Characteristics of Mortar and Concrete under Restrained*
63 *Shrinkage*). Accordingly, in restrained ring test, the effects of specimen size/geometry
64 [11-13]; and moisture gradients and drying condition [12-14] on cracking of concrete
65 or other cement-based materials have been studied. Moreover, novel elliptical ring
66 geometries have been adopted to replace circular ring geometries for investigating

67 shrinkage cracking of mortar and concrete under restrained condition[15-19]. It is
68 generally regarded that, due to stress concentration caused by geometry effect, cracks
69 initiate earlier in elliptical ring specimens than in circular ones though this has yet been
70 validated in those studies. In contrast with the fact that a crack may initiate anywhere
71 in a circular ring specimen, it tends to initiate at certain position in an elliptical ring
72 specimen for a given ring geometry. Therefore, elliptical ring specimens have been
73 employed for assessing cracking tendency of cement-based materials as an improved
74 ring test by some researchers [15-19].

75 On the other hand, in order to study the mechanism of concrete cracking in
76 restrained circular ring specimens, many theoretical methods have been proposed to
77 predict cracking age of concrete. These methods can be classified as stress analysis
78 approach [12-14, 20-22] and fracture resistance curve approach [9, 11, 23, 24]. In
79 conventional stress analysis approach, cracking age could be obtained by comparing
80 the residual stress in a concrete ring and its tensile strength. However, some researchers
81 [9, 11, 23, 24] pointed out that tensile strength-based failure criterion might not yield
82 accurate results for cracking of concrete at early ages, and a fracture-based failure
83 criterion could appear more appropriate. In this case, the fracture resistance curve
84 approach, which is based on fracture mechanics concepts with consideration of the
85 energy balance, has been widely used by many researches [9, 11, 23-28]. By comparing
86 the energy release rate (R -curve) with crack driving energy (G -curve), the onset of crack
87 propagating unsteadily in a restrained concrete ring can be determined. It should be
88 noted that the two methods mentioned above have also been used for predicting
89 cracking ages of concrete in restrained elliptical ring tests [29, 30].

90 Although the fracture resistance curve approach provides a better explanation for what
91 observed in restrained circular/elliptical ring test, it was generally used to predict the

92 critical cracking condition in rings subject to restrained shrinkage. Meanwhile, in
93 restrained circular/elliptical ring test, the age of cracking can be detected from the
94 sudden drop to almost zero in the measured strain of the inner restraining steel ring.
95 Further, the relationship of steel strain and actual residual stress in circular ring test was
96 derived to assess the risk for cracking [31] and to determine the crack growth status in
97 fiber concrete [32]. Generally speaking, concrete is regarded as quasi-brittle material,
98 and there are three distinguished stages in crack propagation process of concrete, i.e.
99 crack initiation, stable propagation and unstable propagation. Restraint disappearing
100 from a steel ring does not suggest the whole crack growth process, i.e. crack initiates
101 and propagates throughout a concrete ring wall.

102 In line with this, the objective of this paper is to investigate crack initiation and
103 propagation process of concrete under a restrained shrinkage condition, demonstrating
104 the failure mechanism and presenting the quantitative assessment of cracking resistance
105 for concrete in the ring test. By introducing the fictitious crack model [33], the nonlinear
106 properties of concrete, characterized by the cohesive force acting on the fictitious crack,
107 is considered. The existence of the cohesive effect reflects the strain localization and
108 nonlinear properties of quasi-brittle materials like concrete, which is an essential
109 difference between quasi-brittle and brittle materials. Meanwhile, by establishing the
110 equilibrium relationship between the crack driving effect caused by the drying
111 shrinkage and the cracking resistance caused by the quasi-brittle properties of concrete,
112 the crack development and evolution processes are elaborated. Further, the difference
113 between the cracking resistance of concrete, which is represented by the crack initiation,
114 and the structural response under a time-dependent drying stimulation, represented by
115 the unstable crack propagation, is clarified. It is expected that the nonlinear fracture
116 mechanics-based numerical model developed in this study will be helpful in exploring

117 the mechanism of restrained circular/elliptical concrete ring test which can be employed
118 for assessing the cracking tendency of concrete and other cement-based materials.

119 Firstly, a numerical approach was developed in this paper to simulate the shrinkage
120 behavior of concrete in restrained ring specimens subject to drying from their outer
121 circumferential cylindrical surface. A fictitious temperature field, which is derived
122 based on free shrinkage data of concrete prisms, was applied on concrete ring
123 specimens in numerical analyses to simulate the mechanical effect of concrete
124 shrinkage on rings under restrained condition. A crack propagation criterion based on
125 the nonlinear fracture mechanics is introduced to determine the crack propagation, in
126 which crack begins to propagate when the stress intensity factors (SIF) K_I^s caused by
127 the applied load (i.e., shrinkage effect in case of concrete rings under restrained
128 condition) exceeds K_I^c caused by the cohesive stress. Using the proposed method, the
129 whole fracture process including crack initiation, stable and unstable propagation is
130 simulated, and the SIF variations at the different crack propagation stages are analyzed.

131

132 **2 Experimental Program**

133

134 The mix proportion for the concrete used for this study was 1:1.5:1.5:0.5 (cement:
135 sand: coarse aggregate: water) by weight, and the maximum of aggregate size was 10
136 mm.

137

138 **2.1 Materials Properties**

139 Mechanical properties of concrete, such as elastic modulus E , splitting tensile
140 strength f_t and uniaxial compressive strength f_c , at different ages were measured using
141 100 mm diameter and 200 mm height cylindrical specimens in this study. After curing

142 in sealed cylindrical moulds in normal laboratory environment for 24 h, the cylindrical
143 specimens were de-moulded and moved into an environment chamber with 23°C and
144 50% relative humidity (RH) for curing till the desirable ages of testing.

145 Regression analyses were conducted on the experimental data to obtain continuous
146 functions that can represent the age-dependent mechanical properties, in this case, E
147 and f_t , for the concrete. It was found that elastic modulus, E , of the concrete at early
148 ages can be predicted using Equation 1.

149

$$150 \quad E(t)=0.0002t^3-0.0134t^2+0.3693t+12.715 \quad (t \leq 28) \quad (1)$$

151

152 Splitting tensile strength, f_t , can be predicted using Equation 2.

$$153 \quad f_t(t)=1.82t^{0.13} \quad (t \leq 28) \quad (2)$$

154

155 In both equations, t is the age (unit: day) of concrete. The values of E and f_t for
156 concrete at other ages which were not directly measured can be obtained from Eqs. (1)
157 and (2).

158

159 ***2.2 Free Shrinkage Tests***

160 Free shrinkage was measured on concrete prisms with the dimensions of 280 mm
161 in length and 75 mm square in cross section, conforming to ISO 1920-8, subject to
162 drying in the same environment as for curing concrete cylinders. Their longitudinal
163 length change was monitored by a dial gauge, which was then converted into shrinkage.
164 Considering that concrete shrinkage depends on the exposed surface area-to volume
165 (A/V) ratio, four different exposure conditions, i.e. four different values of A/V , were

166 investigated in this study. They are all surfaces sealed, all surface exposed, two side
167 surfaces sealed and three side surfaces sealed, respectively. It should be noted that side
168 surface refers to the surface with the dimensions of $280 \times 75 \text{ mm}^2$. In experiment,
169 double-layer aluminum tape was used to seal the desired surfaces which were not
170 intended for drying. A two side surfaces sealed prism in free shrinkage test is shown in
171 Figure 1. Initial measurement was carried out immediately after the concrete prisms
172 were de-molded at the age of 1 day and the measurements were continuously recorded
173 twice per day until 28 days. Figure 2 shows the measured free shrinkage strain of
174 concrete at various ages under different exposure conditions.

175 It should be noted that the computation of free shrinkage strain in a concrete ring is an
176 approximate estimation for the purpose of simplification, which match the geometry of
177 the prism to the ring through the ratio of A/V . A more complex and accuracy approach
178 was introduced in Reference [28]. The free shrinkage prisms had an identical volume
179 to surface ratio to that of a short concrete ring with the depth of 75 mm when drying
180 from the top and bottom surfaces. Therefore a direct correlation can be made with the
181 shrinkage properties of the ring. In the case of tall rings drying from outer surface, the
182 shrinkage strain of concrete can be derived by introducing the average humidity profiles
183 and the relationship between shrinkage and relative humidity.

184

185 ***2.3 Restrained ring test***

186 In order to investigate the effects of the inner steel core on cracking age of
187 restrained concrete rings and the mechanism of restrained ring test, two kinds of steel
188 core, which are circular and elliptical, were investigated in current experiment program.

189 A circular concrete ring with its inner diameter is denoted as R_0 (see Figure 3a), and an
190 elliptical concrete ring, with the major and minor semi-axes of its inner circumference
191 is denoted as R_1 and R_2 , respectively (see Figure 3b). According to the study of Zhou et
192 al. [30], compared with traditional circular concrete rings, elliptical concrete rings with
193 R_1/R_2 between 2 and 3 can provide higher degree of restraint leading to shorter cracking
194 period in restrained shrinkage ring test so that to accelerate ring test. Therefore, in this
195 study, for the concrete elliptical ring specimens with a 37.5 mm-thick wall same as that
196 recommended by ASTM C1581/C1581M-09a, the inner major radius, R_1 , was chosen
197 as 150 mm and the inner minor radius, R_2 , as 75 mm while the radius, R_0 , of the inner
198 circumference of the circular rings was designed as 150 mm same as R_1 . The wall
199 thickness of restraining steel cores and the height of the specimens were as 12.5 and 75
200 mm, respectively. Following ASTM C1581/C1581M-09a protocol, the top and bottom
201 surfaces of ring specimens were sealed using two layers of aluminum tape and drying
202 was only allowed through the outer circumferential cylindrical surface of the concrete
203 rings. The test setup and sealed specimens are shown in Figure 4. The strain gauges
204 were then connected to the data acquisition system, and the instrumented ring
205 specimens were finally moved into an environmental chamber after the first day curing
206 for continuous drying under the temperature 23°C and RH 50% till the first crack
207 occurred.

208 It should be noted that, according to Radlinska et al. [34], specifying the precision
209 of the restrained ring test in terms of standard deviation of measured strain is more
210 promising than of the age of cracking. It is because the variability in the cracking age
211 shows time dependence and it is much higher for the cracking occurred at later age.
212 However, in this study, the cracking age was used to reflect the time-dependent material
213 properties rather than the variability of the ring test. Therefore, it is still employed in

214 this study to compare the cracking tendency of concrete in elliptical and circular ring
215 tests.

216

217 **3 Numerical Modeling**

218

219 *3.1 Modeling of Restrained Shrinkage*

220 In current research, a derived fictitious temperature field is applied to concrete to
221 represent the mechanical shrinkage effect so that a combined thermal and structural
222 analysis can be adopted to analyze cracking in a concrete ring specimen caused by
223 restrained shrinkage. The restrained circular ring test is recommended by AASHTO and
224 ASTM, for assessing the cracking tendency of concrete under a restrained drying
225 shrinkage condition. However, in the case of concrete deformation caused by large
226 temperature variation, the conventional ring test cannot serve the purpose because the
227 restraining effect varies with the steel ring dimensions caused by varying temperature.
228 Particularly, the restraint will disappear if the concrete expansion occurs under elevated
229 temperature. To overcome these limitations, the dual ring test was proposed by Schlitter
230 et al. [35-37], in which invar with a very low thermal expansion coefficient was used
231 to make the central restraining steel ring. Meanwhile, the autogenous shrinkage can be
232 reduced or eliminated by controlling the temperature reduction in a concrete ring, so
233 that the remaining stress capacity of concrete can be better assessed. The research
234 conducted by Schlitter et al. [35-37] extends the application of the restrained ring test
235 and provide an improved understanding of the cracking potential of a cement mixture.
236 However, it should be noted that the fictitious temperature drop proposed in this study
237 is to numerically simulate the concrete shrinkage effect under drying, as opposed to a
238 real temperature drop experienced in a ring test. For the purpose of the analyses, there

239 is no fictitious temperature drop enforced on the central restraining steel ring therefore
240 it makes no difference whether traditional steel or invar steel is used for making the
241 central restraining steel ring.

242 With the implementation of the fictitious temperature field, shrinkage of concrete
243 caused by the temperature field is restrained by the inner steel core, resulting in
244 compressive stress developed in the steel core and tensile stress in the concrete ring.
245 The derivation of the fictitious temperature field is elaborated elsewhere [30]. As the
246 result of this exercise, Figure 5 presents the derived relationship between fictitious
247 temperature drop and A/V ratio at 2 days interval for a concrete element irrespective of
248 its geometry/shape. It should be noted that although Fig. 5 only presents the curves at
249 2 days interval, fictitious temperature drop was actually calculated for each day which
250 was then used to update the input data for FE analyses of concrete rings in this study.
251 For a given concrete ring with certain exposure condition (i.e. certain A/V ratio), the
252 relationship between fictitious temperature drop and concrete age can be derived by
253 linear interpolation from the relationship between A/V ratio and concrete age obtained
254 in Figure 5.

255

256 ***3.2 Crack initiation and propagation***

257 According to experimental observation, in case of a thick ring (i.e., the ring wall
258 thickness is 75 mm), cracking initiates at the outer circumferential surface of a
259 restrained concrete ring, which accords well with the findings of Hossain and Weiss
260 [13] by acoustic emission testing. In case of a thin ring, to the best of the authors'
261 knowledge, there is no explicit experimental evidence that can support whether a crack
262 initiates at its outer or inner circumferential surface. Moon and Weiss [12] studied the
263 residual stress distribution in both thick and thin circular rings by considering the

264 moisture gradient across a concrete ring wall. Based on their results, tensile stress is the
265 highest on the outer circumference of a concrete ring. Due to this, cracking always
266 initiates at the outer circumference surface of a concrete ring when drying from its outer
267 circumferential surface, no matter how thick a concrete wall is if the moisture gradient
268 is taken into account in analyses. It is because that the moisture gradient is very
269 significant with the majority of the drying taking place from the outer circumference of
270 a concrete ring at early ages (i.e. low value of γ , where γ is the product of the moisture
271 diffusion coefficient and drying time). Under this condition, the crack in a restrained
272 concrete ring will be mainly caused by the self-shrinkage due to the moisture gradient
273 rather than from restraining effect from the central steel ring. However, it makes sense
274 that the crack initiation is resulted from the restraining-dominated effect from the
275 central steel ring in case of a thin-wall concrete ring. In which case, the crack will
276 initiate at the inner circumference of a concrete ring. Meanwhile, by considering that a
277 thin ring has a larger A/V ratio than a thick one when drying from its outer
278 circumference surface, the moisture gradient across a thin ring wall can be ignored and
279 the approximate uniform shrinkage across it is assumed. However, it should be noted
280 that it is necessary to further examine the initial crack position of a thin ring under outer
281 circumferential surface drying from experiment.

282 In this study, cracks are assumed to initiate when the maximum circumferential
283 tensile stress of concrete exceeds its tensile strength f_t . According to the experimental
284 and numerical results conducted by Zhou et al. [30], crack randomly initiates at the
285 inner circumference of a circular ring specimen. While in the case of an elliptical ring
286 specimens with $R_1=150$ mm, $R_2=75$ mm, crack initiates close to the vertices on the
287 major axis of the inner elliptical circumference (See Figure 6). Therefore, after crack
288 initiation, a pre-crack, with the length of 2 mm, is set at the position where the

289 maximum circumferential tensile stress occurs. In order to consider the softening
290 behavior in micro-cracks, the fictitious crack model [33] is introduced in the fracture
291 analysis through establishing softening stress (σ)-crack opening displacement (w)
292 relationship of concrete. In this paper, the bilinear expression for σ - w (see Figure 7) is
293 chosen in the proposed numerical approach which is presented as follows:

294 According to Peterson [38], σ_s , w_s and w_0 can be determined as follows:

$$295 \quad \sigma_s = f_t / 3 \quad (3)$$

$$296 \quad w_s = 0.8G_F / f_t \quad (4)$$

$$297 \quad w_0 = 3.6G_F / f_t \quad (5)$$

298 Where w_0 is the displacement of the terminal point of σ - w curve beyond which no
299 stress can be transferred, i.e. the stress-free crack width, w_s and σ_s is the displacement
300 and stress, respectively, corresponding to the break point in the bilinear σ - w relationship.
301 These parameters and the σ - w relationship can be derived given the fracture energy G_F
302 and the tensile strength f_t . Here, f_t is obtained from Equation (2) and G_F is from the
303 formula recommended by CEB-FIP model code 2010.

304 Further, a crack propagation criterion based on nonlinear fracture mechanics is
305 employed for predicting cracking propagation process in circular/elliptical concrete
306 rings under restrained shrinkage. According to this criterion, when the SIF caused by
307 deriving forces exceeds the one by cohesive forces, i.e. $K_I^s - K_I^\sigma \geq 0$, crack will propagate.
308 It represents the competition between the crack driving forces which attempt to open
309 the crack and the cohesive forces which attempt to close the crack [39]. This criterion
310 has been successfully used to simulate the crack propagation in reinforced concrete [40],
311 mode-I and mixed-mode fracture [41, 42] and multiple cohesive crack propagation [43]
312 in concrete. In this study, SIFs caused by deriving forces, i.e. shrinkage effect, and
313 cohesive force are denoted as K_I^s and K_I^σ , respectively. Moreover, in this study, the age-

314 dependent effective elastic modulus of concrete adopted in numerical analyses was
315 taken as 60% of the value obtained from Equation (1) to account for creep effects.
316 Similar measure in taking into account creep effect by reducing elastic modulus was
317 also taken by [14, 30] when analyzing cracking in circular/elliptical concrete rings
318 under restrained shrinkage. In numerical analysis, singular element was used to
319 calculate SIF at the tip of crack. In order to eliminate the effect of friction between
320 concrete and steel, the outer circumferential surface of the steel ring, which contacts the
321 inner circumferential surface of the concrete ring, was coated with a release agent as
322 suggested by ASTM C1581/C1581M-09a when preparing ring tests. Accordingly, in
323 numerical analyses, contact element with zero friction between the contact pair was
324 utilized to simulate this measure in conducting concrete ring tests.

325 In summary, the following steps were taken in analyzing crack initiation and
326 propagation of a concrete ring subject to restrained shrinkage:

327

- 328 (1) Measure mechanical properties, including f_t and E , of concrete and free
329 shrinkage through a series of concrete prisms with different A/V ratios at
330 various ages. Convert the results of free shrinkage into a relationship between
331 fictitious temperature drop and A/V ratio for a concrete element at various ages.
- 332 (2) Derive the fictitious temperature drop for a circular/elliptical ring specimen at
333 various ages by linear interpolation from the relationship between fictitious
334 temperature drop and A/V ratio obtained in step (1).
- 335 (3) Conduct an elastic analysis in a concrete ring by applying the corresponding
336 fictitious temperature drop obtained in Step (2) on it, until the maximum
337 circumferential tensile stress of concrete exceeds its tensile strength. Pre-set a 2

338 mm long fictitious crack on the location where the maximum circumferential
339 tensile stress occurs in a circular/elliptical concrete ring.

340 (4) Apply cohesive stress on the fracture process zone (FPZ) according to the σ - w
341 relationship. Calculate SIFs at crack tip caused by the cohesive force and
342 shrinkage effect at various ages until K_1^s exceeds K_1^σ .

343 (5) Add an increment of crack length $\Delta a=2$ mm and repeat Step (4) and (5) until
344 crack propagates throughout the section of the concrete ring wall.

345

346 **4 Results and discussions**

347

348 According to the numerical analysis results of Zhou et al. [30], the maximum
349 circumferential tensile stress of concrete exceeds its tensile strength at the ages of 12
350 days for elliptical rings with $R_1=150$ mm and $R_2=75$ mm. In the case of circular rings
351 with $R_0=150$ mm, the cracking age is 15 days. After crack initiation, a pre-crack is
352 located on inner circumference of a circular ring, and the pre-crack is close to the
353 vertices on the major axis of the inner circumference for an elliptical ring. There is
354 cohesive stress acting on the pre-crack, which can be determined by the σ - w relationship
355 specified in Figure 7. Figure 8 illustrates the pre-crack locations in circular and elliptical
356 rings adopted in this study.

357 In order to elaborate crack propagation after initiation, Figures 9 and 10 present
358 the evolution of SIF caused by shrinkage effect and cohesive stress in circular/elliptical
359 concrete rings. For the circular ring with $R_0=150$ mm at the cracking age of 15 days,
360 the SIFs at the tip of 2 mm pre-crack satisfy the criterion of $K_1^s - K_1^\sigma=0$, which suggests
361 that the crack driving force caused by shrinkage effect can overcome the resistance
362 caused by cohesive force. So the crack can propagate further under that condition.

363 While, $K_I^s - K_I^\sigma < 0$ (See Point A in Figure 9a) when the crack propagates to the length of
364 4 mm, it indicates that the resistance exceeds the crack driving force and the crack
365 cannot propagate further. So in order that the crack is able to move forward, it is
366 necessary to enhance the shrinkage effect of concrete, i.e. the crack driving energy.
367 When concrete gets mature, i.e., from the 15th to the 16th day, the relationship between
368 K_I^s and K_I^σ evolves to $K_I^s - K_I^\sigma > 0$ corresponding to a crack length of $a=4$ mm (See Point
369 B in Figure 9b). Moreover, at the age of 16 days, K_I^s becomes always greater than K_I^σ
370 and the difference between K_I^s and K_I^σ (i.e. $K_I^s - K_I^\sigma$) keeps increasing with the increase
371 of crack length. This demonstrates that the crack driving force exceeds the resistance
372 which is the cohesive stress tending to close the crack and the crack will propagate
373 throughout the concrete ring wall at the 16th day. Comparing with the scenario of
374 monotonic increasing of K_I^s with the increase of crack length, there are three
375 distinguished stages in the evolution of K_I^σ , i.e., monotonically increasing before crack
376 length reaches 14 mm (i.e. $a \leq 14$ mm), keeping plateaued when crack length is between
377 14 and 24 mm (i.e. $a = 4 \sim 24$ mm), and monotonically decreasing when crack length
378 exceeds 24 mm (i.e. $a > 24$ mm). The variation of K_I^σ can be explained by the shortening
379 of FPZ when the crack tip is closed to the outer circumferential surface of a concrete
380 ring.

381 For the elliptical ring with $R_1=150$ mm and $R_2=75$ mm, when crack initiates at the
382 age of 12 days, $K_I^s - K_I^\sigma = 0$ corresponding to a crack length $a=2$ mm. It indicates that the
383 crack can continue propagating under that condition. In numerical analysis, a new crack
384 length in this case 4 mm is reached by giving a crack length increment $\Delta a=2$ mm to the
385 original crack length 2 mm. However, when crack propagates to 4 mm long, it reaches
386 a different scenario. In this case, SIF caused by cohesive force, i.e. K_I^σ , becomes greater
387 than that caused by the driving force, i.e. K_I^s , which in fact is due to shrinkage effect. It

388 indicates that the crack cannot propagate further until the driving force can be enhanced
389 and SIF caused by driving force, i.e. K_I^σ , becomes greater than SIF caused by cohesive
390 force, i.e. K_I^σ . As concrete gets mature as drying continues, the accumulative shrinkage
391 effect keeps increasing. At the age of 13 days, K_I^s eventually becomes greater than K_I^σ
392 and the deviation between K_I^s and K_I^σ becomes greater than 0 thereafter. Therefore the
393 crack will keep propagating till throughout the concrete ring wall.

394 Besides, before crack starts unstable propagation, the maximum value of $K_I^s - K_I^\sigma$
395 occurs at the crack length of $a=4$ mm for a circular ring (See Point A in Figure 9a), and
396 $a=8$ mm for an elliptical ring (See Point A in Figure 10a). Within one day after crack
397 initiates, it can propagate throughout the concrete ring wall. At the corresponding points
398 B in Figures 9b and 10b, K_I^s becomes greater than K_I^σ . Therefore, it can be concluded
399 that the crack lengths at the onset of unstable propagation for circular and elliptical
400 rings are 4 and 8 mm, respectively. The difference in crack length at the onset of
401 unstable propagation in circular and elliptical concrete rings is due to the difference in
402 stress gradient across a concrete ring wall with circular and elliptical geometries.
403 Comparing with a circular ring geometry, the stress gradient in a concrete wall is more
404 significant for an elliptical ring geometry [30]. Meanwhile, the stable propagation
405 length gives a reference to determine an appropriate pre-crack length a_0 for numerical
406 analysis of concrete rings under restrained shrinkage. For example, in the case of a
407 circular ring, the stable propagation of concrete crack cannot be captured by numerical
408 analysis once a_0 exceeds 4 mm under current study condition.

409 In the standard restrained ring test methods recommended by AASHTO T334-08
410 and ASTM C1581/C1581M-09a, compressive strain developed in a steel ring caused
411 by shrinkage of mortar or concrete surrounding it is monitored by strain gauges attached
412 on the inner circumferential surface of the steel ring. As concrete is getting mature, it

413 shrinks but is restrained by the inner steel ring. Therefore, tensile stress develops in the
414 concrete ring and compression stress in steel ring. Cracking of the concrete ring is
415 indicated by a sudden drop in the steel ring strain. However, it should be noted that the
416 sudden drop of steel ring strain only represents the release of restraint effect in concrete
417 caused by the inner steel ring, rather than explicitly reflects the statue of crack, i.e.
418 initiation, stable propagation or unstable propagation, in concrete. According to the
419 numerical results conducted in this study, for the purpose of crack propagation, the
420 restraining effect should be strengthened after crack initiation until the driving force
421 caused by restraint shrinkage becomes greater than the resistance force so that crack
422 can continue growing till propagating throughout the concrete ring wall. Therefore, the
423 sudden drop of steel ring strain observed in the test recommended by ASTM
424 C1581/C1581M-09a illustrates the crack unstable propagation, which is regarded as a
425 structural response, rather than the crack initiation, which is regarded as a crack
426 resistance of concrete as a material property. Structural response of a concrete element
427 does depend on its thickness. In this case, the cracking age of a restrained concrete ring
428 depends on its ring wall thickness. However, it should be noted that the period from
429 crack initiation to unstable propagation is very short, which is within one day, in both
430 circular and elliptical rings subject to restrained shrinkage for the cases investigated in
431 this study. Moreover, although K_1^s is smaller than K_1^σ after crack initiation, the
432 difference between K_1^s and K_1^σ is very marginal with the maximum value of 0.08
433 MPa·mm^{1/2} in a circular ring (See Point A in Figure 9a) and 0.38 MPa·mm^{1/2} in an
434 elliptical ring (See Point A in Figure 10a). These differences can be ignored compared
435 with the SIFs caused by restrained shrinkage effect, which are about 6.5 MPa·mm^{1/2} in
436 both geometries. Therefore, it is reasonable to accept that restrained shrinkage ring test
437 can approximately determine crack resistance of concrete and other cement-based

438 materials as per AASHTO T334-08 and ASTM C1581/C1581M-09a. According to
439 numerical analysis results, the cracking ages for circular and elliptical rings are 16 and
440 13 days, respectively, which are the age at the onset of crack unstable propagation. The
441 experimental results of cracking ages with respect to the circular and elliptical rings
442 are about 15 and 10 days, which are determined by a sudden drop in the steel ring strain.
443 The cracking ages predicted through the numerical model agreed reasonably well with
444 experimental results indicating that the proposed numerical model is reliable.

445 Further, it is well known that crack initiation in concrete can be regarded as
446 material behavior on cracking resistance, which is independent of specimen geometry
447 and boundary condition. However, unstable concrete crack propagation in a restrained
448 concrete ring represents structural response to external stimulation, i.e. it is structure
449 behavior, which depends on specimen geometry and boundary condition. In restrained
450 shrinkage ring test, concrete crack actually experiences the full spectrum of initiation,
451 stable propagation and unstable propagation till propagating throughout the ring wall.
452 The observed sudden strain drop of an inner steel ring reports that a crack propagates
453 throughout a concrete ring wall, i.e. unstable propagation. Then, the results obtained in
454 the ring test indicate structural behavior of a concrete ring subject to restrained
455 shrinkage, rather than material behavior of concrete as per the suggestions of the
456 AASHTO T334-08 and ASTM C1581/C1581M-09a. It is worthy pointing out here that,
457 from the point of view of applying the ring test to assess cracking tendency of concrete
458 as a material property of concrete, the difference in ages at crack initiation (representing
459 material behavior) and unstable propagation (representing structural behavior) is very
460 marginal although the mechanical mechanism is distinguished.

461

462 **5 Conclusions**

463

464 In this study, crack initiation and propagation in restrained concrete rings, both circular
465 and elliptical, is analyzed. A fictitious temperature drop is applied on concrete to
466 simulate the shrinkage effect of concrete due to drying. Combined with the fictitious
467 crack model, a numerical mode is established for investigating crack initiation and
468 propagation in circular/elliptical concrete rings under restrained shrinkage for the
469 purpose of assessing cracking tendency of concrete and other cement-based materials
470 as per AASHTO T334-08 and ASTM C1581/C1581M-09a. The evolution of SIF
471 caused by restrained shrinkage and cohesive force for circular and elliptical rings is
472 investigated with respect to concrete age. Based on the results presented in this paper,
473 the following conclusions can be drawn.

474 (1) The initial cracking ages predicted through the numerical model agree reasonably
475 well with experimental results indicating that the proposed numerical model is
476 reliable. The fracture-based method developed in this study can be employed for
477 simulating crack propagation process in circular/elliptical concrete rings under
478 restrained shrinkage.

479 (2) There is a stable crack propagation process after crack initiation in restrained
480 circular/elliptical concrete rings. The sudden strain drop measured from the internal
481 surface of a restraining steel ring in a standard ring test indicates that a crack
482 propagates throughout the concrete ring wall. For both circular and elliptical rings
483 in the cases studied of this paper, the crack initiation and propagation throughout
484 the concrete ring wall do not occur at the same time. Concrete cracks can propagate
485 through a ring wall within one day after initiation. However, it should be noted here
486 that the time, from crack initiation to propagation throughout a concrete ring wall,

487 depends on the type of concrete and the geometry of a ring specimen. In certain
488 cases, this can be much longer than 1 day.

489 (3) The cracking age of concrete or other cement-based materials, which is
490 characterized as a crack propagating throughout the concrete ring wall in a
491 restrained ring test, depends on the degrees of restraint, resulted from the properties
492 of steel and concrete (i.e. elastic modulus, Poisson's ratio etc.), and the geometry
493 of the ring specimen (i.e. radius of steel ring, thickness of steel and concrete rings,
494 etc.) [14]. So results given by the AASHTO/ASTM restrained ring test actually
495 reflects the response of a concrete ring as a structure to external stimulation, in this
496 case restrained concrete shrinkage. Thus the AASHTO/ASTM restrained ring test
497 actually provides information about the structural behavior of a restrained concrete
498 ring, rather than material behavior of concrete as per the suggestions of the
499 AASHTO T334-08 and ASTM C1581/C1581M-09a.

500

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506

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617 **Figure captions.**

618 Figure 1. Free shrinkage test setup

619 Figure 2. Shrinkage strain of concrete obtained from free shrinkage test

620 Figure 3. Notation of geometrical parameters; **a** circular ring, **b** elliptical ring

621 Figure 4. Restrained shrinkage ring test; **a** test setup for an elliptical ring, **b** sealed
622 specimens

623 Figure 5. Derived fictitious temperature drop with respect to A/V ratio for a concrete
624 element

625 Figure 6. Crack position in an elliptical ring specimen

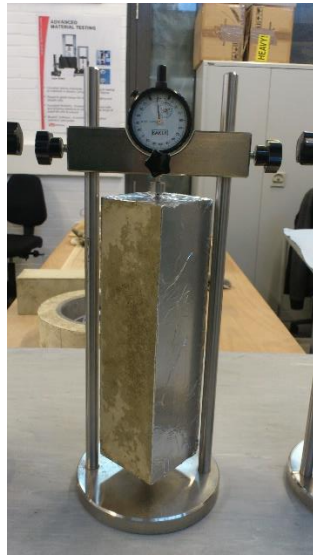
626 Figure 7. Bilinear σ - w softening curve for concrete

627 Figure 8. Pre-crack in concrete rings; **a** circular, **b** elliptical

628 Figure 9. Evolution of SIFs in a circular ring at the ages of crack initiation and unstable
629 propagation; **a** 15 days, **b** 16 days

630 Figure 10. Evolution of SIFs in an elliptical ring at the ages of crack initiation and
631 unstable propagation; **a** 12 days, **b** 13 days

632

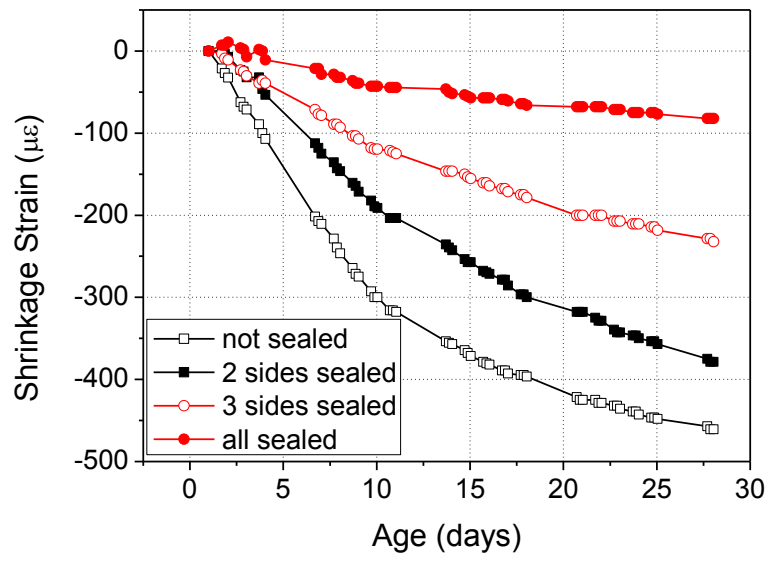


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Fig. 1. Free shrinkage test setup

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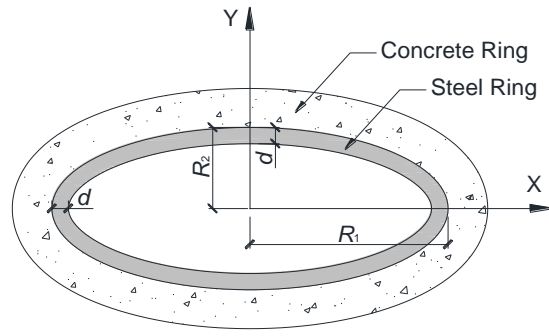
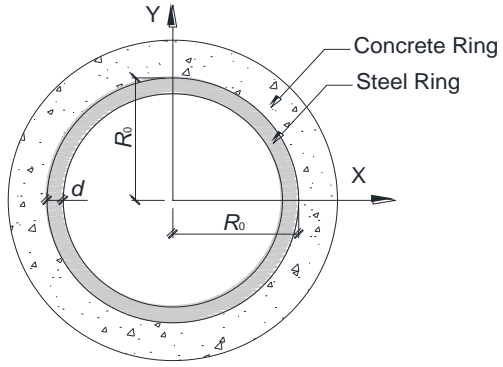


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Fig. 2. Shrinkage strain of concrete obtained from free shrinkage test

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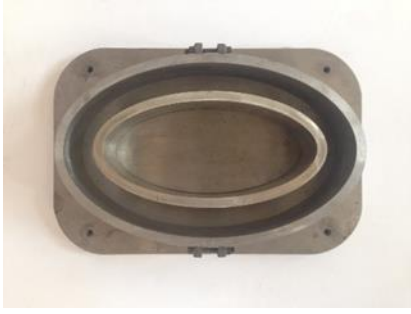
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(a)

(b)

Fig. 3. Notation of geometrical parameters; **a** circular ring, **b** elliptical ring



(a)



(b)

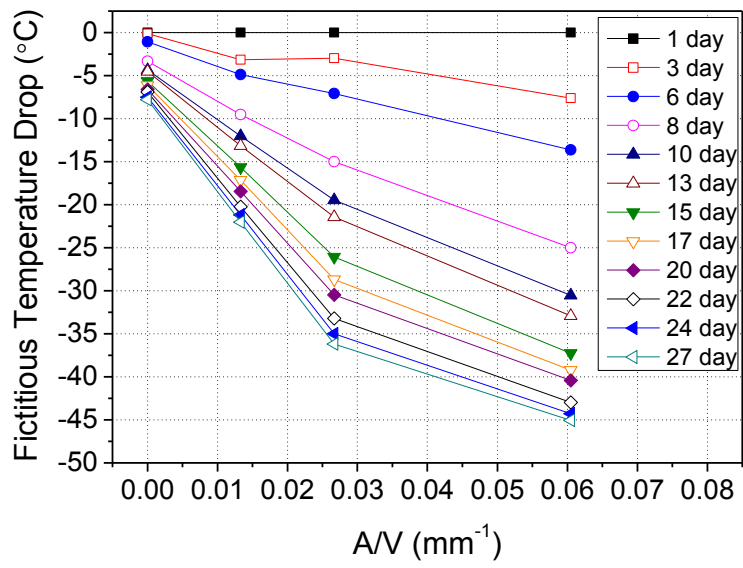
Fig. 4. Restrained shrinkage ring test; **a** test setup for an elliptical ring, **b** sealed specimens

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648 **Fig. 5.** Derived fictitious temperature drop with respect to A/V ratio for a concrete element

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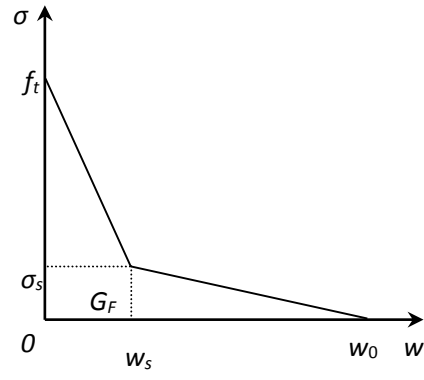


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Fig. 6. Crack position in an elliptical ring specimen

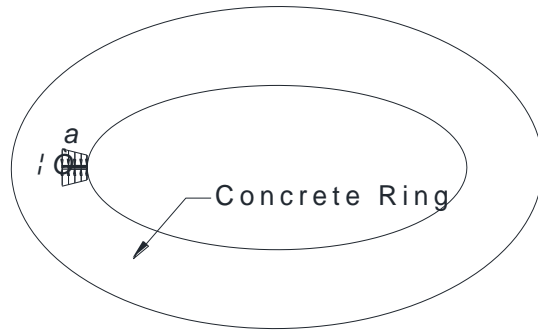
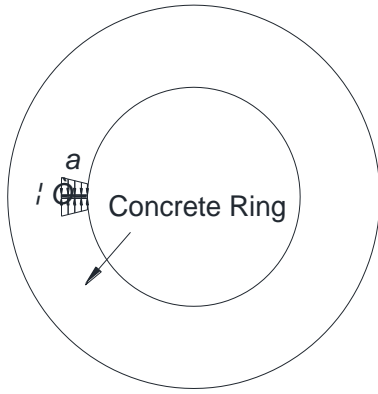


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Fig. 7. Bilinear σ - w softening curve for concrete

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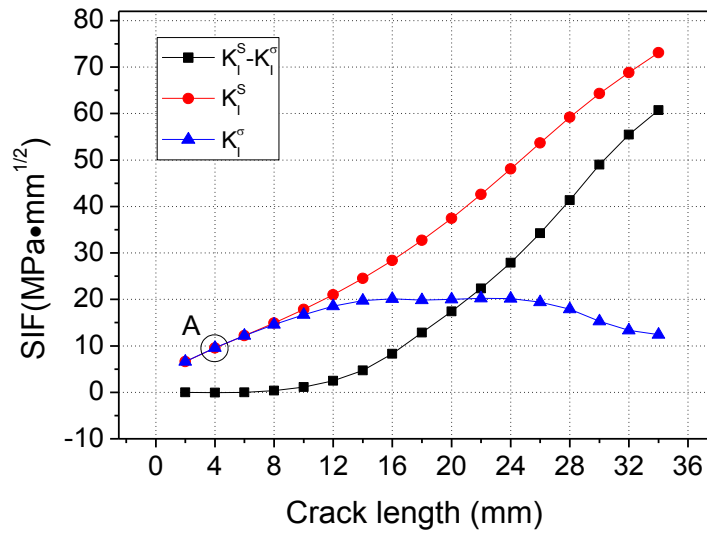
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(a)

(b)

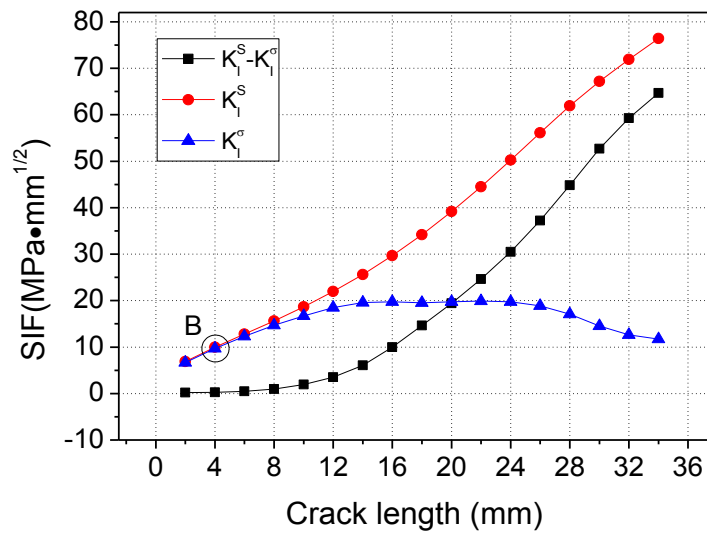
Fig. 8. Pre-crack in concrete rings; **a** circular, **b** elliptical



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(a)



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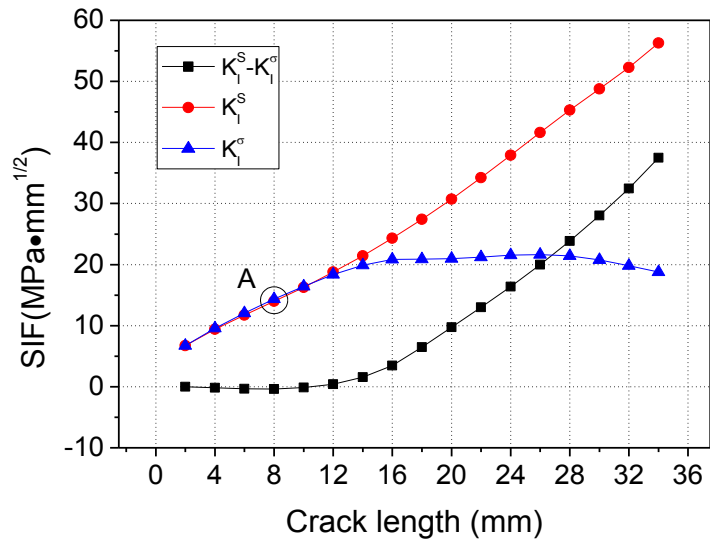
664 **Fig. 9.** Evolution of SIFs in circular ring at the ages of crack initiation and unstable propagation; **a** 15

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days, **b** 16 days

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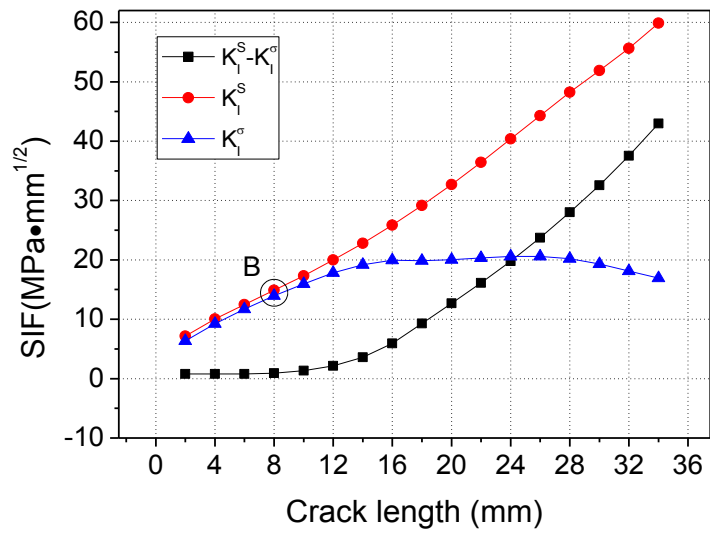
(b)



667

668

(a)



669

670

(b)

671 **Fig. 10.** Evolution of SIFs in elliptical ring at the ages of crack initiation and unstable propagation; **a**

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12 days, **b** 13 days