Robust H_∞ Filtering for Stochastic Time-Delay Systems With Missing Measurements

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***Abstract*—In this paper, the robust H_∞ filtering problem is studied for stochastic uncertain discrete time-delay systems with missing measurements. The missing measurements are described by a binary switching sequence satisfying a conditional probability distribution. We aim to design filters such that, for all possible missing observations and all admissible parameter uncertainties, the filtering error system is exponentially mean-square stable, and the prescribed H_∞ performance constraint is met. In terms of certain linear matrix inequalities (LMIs), sufficient conditions for the solvability of the addressed problem are obtained. When these LMIs are feasible, an explicit expression of a desired robust H_∞ filter is also given. An optimization problem is subsequently formulated by optimizing the H_∞ filtering performances. Finally, a numerical example is provided to demonstrate the effectiveness and applicability of the proposed design approach.**

***Index Terms*—H_∞ filtering, missing measurements, parameter uncertainty, robust filtering, time-delay systems.**

I. Introduction

In most literature concerning filtering techniques, it is implicitly assumed that the measurements always contain consecutive useful signals (see, e.g., [1], [2], [9], and [10]). However, in practical applications such as target tracking, there may be a nonzero probability that any observation consists of noise alone if the target is absent, i.e., the measurements are not consecutive but contain missing observations. The missing observations are caused for a variety of reasons, for example, the high maneuverability of the tracked target, a failure in the measurement, intermittent sensor failures, network congestion, accidental loss of some collected data, or some of the data may be jammed or coming from a very noisy environment, etc. Note that in network signal transmissions, the missing observation is also called dropout or intermittence (see [13] and [22]).

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The filtering problem for systems with missing measurements has received much attention during the past few years. Basically, there have been two ways to model the missing measurement phenomena, i.e., using binary switching sequence and using jump linear systems. The binary switching sequence is specified by a conditional probability distribution and enters into the system observation. It can be viewed as a Bernoulli distributed white sequence taking on values of 0 and 1. Much work has been done on such a model. As early as in [15], the optimal recursive filter was obtained for systems with missing measurement. The results in [15] were generalized in [12] and [16], where the least mean-squared error recursive estimator was investigated over the class of linear filters, when the binary switching sequence was not necessarily independent and identically distributed (i.i.d.). A similar model was employed in [25] to study the filter design problem with error variance constraints. Recently, the finite-horizon robust filtering problem has been considered in [26] for discrete-time stochastic systems with probabilistic missing measurements subject to norm-bounded parameter uncertainties. The statistical convergence properties of Kalman filter with missing measurement have been addressed in [22], and the existence of a critical value has been shown for the arrival rate of the observations. Another way is to model the missing measurement as a Markovian jumping process. The filtering problem with missing measurement has been studied in [4] and [23], and the filters guaranteeing expected estimation error covariance have been designed based on jump Riccati equations. In [18] and [19], an incompleteness matrix has been introduced to quantify the missing data, and the robust filtering problem with missing data has been investigated in terms of a recursive state estimator involving a jump Riccati differential equation and jump state equations.

On the other hand, it has now been well known that time delays exist in many practical systems and are often a primary source of instability and performance degradation. For example, the current networks themselves are dynamic systems and induce possible delays via network communication due to limited bandwidth. Obviously, the network delay and data transmission loss are two main problems that deserve much research attention in networked operation systems. Recently, more and more efforts have been focused on the problem of H_∞ filtering for various time-delay systems, and many approaches have been proposed, including the Riccati equation approach [17], [21], [27], the linear matrix inequality (LMI) approach [5]–[8], [11], [14], [28], and the polynomial equation approach [29]. Unfortunately, up to now, in almost all existing works dealing with the filtering problem for time-delay systems, the measurement missing phenomena have seldom been taken into account. For discrete-time systems in the simultaneous presence of time delays, missing measurements, and parameter uncertainties, the problem of robust H_∞ filtering has not been fully investigated and remains to be challenging.

In this paper, we study the robust H_∞ filtering problem for a class of uncertain discrete time-delay systems with missing measurements. The missing measurement is modeled as a Bernoulli distributed white sequence with a known conditional probability distribution that enters into the observation equation. A sufficient condition for the existence of a feasible solution to the problem is derived, which guarantees that the filtering error system is exponentially mean-square stable and the filtering error satisfies the H_∞ robustness performance constraint, for all possible missing observations and all admissible parameter uncertainties. An LMI approach is developed to design the expected filters, and the effectiveness of the proposed method is illustrated by means of a numerical example.

Notation: The notation used here is fairly standard. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n \times m$ real matrices. The notation $X \geq Y$ (respectively, $X > Y$), where X and Y are symmetric matrices, means that $X - Y$ is positive semidefinite (respectively, positive definite). The superscript "T" denotes the transpose. $\mathbb{E}\{x\}$ stands for the expectation of the stochastic variable x. $\text{Prob}\{\cdot\}$ means the occurrence probability of the event "$\cdot$". If A is a matrix, $\lambda_{\max}(A)$ (respectively, $\lambda_{\min}(A)$) means the largest (respectively, smallest) eigenvalue of A. $l_2[0,\infty)$ is the space of square integrable vectors, and $\mathbb{I}^+$ is the set of positive integer. In symmetric block matrices, "$*$" is used as an ellipsis for terms induced by symmetry. Sometimes, the arguments of a function will be omitted in the analysis when no confusion can arise.

II. Problem Formulation and Preliminaries

Consider the following class of uncertain discrete time-delay systems:

$$\begin{cases} x_{k+1} = (A+\Delta A)x_k + (A_d + \Delta A_d)x_{k-d} + Bw_k \\ z_k = Cx_k + C_d x_{k-d} + Dw_k \\ x_k = \phi_k, \quad k = -d, -d+1, \ldots, 0 \end{cases} \tag{1}$$

where $x_k \in \mathbb{R}^n$ is a state vector; $z_k \in \mathbb{R}^r$ is the signal to be estimated; $w_k \in \mathbb{R}^q$ is the disturbance input belonging to $l_2[0,\infty)$; and A, A_d, B, C, C_d, and D are known real constant matrices with appropriate dimensions. ΔA and ΔA_d are unknown matrices describing parameter uncertainties, the integer $d > 0$ is a time delay, and ϕ_k is a real-valued initial function on $[-d, 0]$.

In this paper, the parameter uncertainties are assumed to be of the form

$$[\Delta A \quad \Delta A_d] = HF_k[E_1 \quad E_2] \tag{2}$$

where H, E_1, and E_2 are known real constant matrices of appropriate dimensions, and F_k represents an unknown real-valued time-varying matrix satisfying

$$F_k F_k^T \leq I. \tag{3}$$

The measurements, which may contain missing data, are described by

$$y_k = \gamma_k C_2 x_k + D_2 w_k \tag{4}$$

where the stochastic variable $\gamma_k \in \mathbb{R}$ is a Bernoulli distributed white sequence taking the values of 0 and 1 with

$$\text{Prob}\{\gamma_k = 1\} = \mathbb{E}\{\gamma_k\} := \beta \tag{5}$$
$$\text{Prob}\{\gamma_k = 0\} = 1 - \mathbb{E}\{\gamma_k\} := 1-\beta \tag{6}$$

and $\beta \in \mathbb{R}$ is a known positive scalar. $y_k \in \mathbb{R}^p$ is the measured output vector, w_k is defined in (1), and C_2 and D_2 are known real constant matrices of appropriate dimensions.

Remark 1: The system measurement mode (4), which can be used to represent missing measurements or uncertain observations, was first introduced in [15] and has been subsequently studied in many papers (see, e.g., [16], [22], [25], and [26]).

Consider the following filter for system (1) and (4):

$$\begin{cases} \hat{x}_{k+1} = G\hat{x}_k + K(y_k - \beta C_2 \hat{x}_k) \\ \hat{z}_k = L\hat{x}_k \end{cases} \tag{7}$$

where $\hat{x}_k$ is the state estimate, $\hat{z}_k$ is an estimate for z_k, and G, K, and L are filter parameters to be determined.

By defining

$$\eta_k = \begin{bmatrix} x_k \\ \hat{x}_k \end{bmatrix}, \quad \bar{A} = \begin{bmatrix} A & 0 \\ \beta K C_2 & G - \beta K C_2 \end{bmatrix}, \quad \tilde{H} = \begin{bmatrix} H \\ 0 \end{bmatrix},$$
$$\tilde{E}_1 = [E_1 \quad 0], \quad \bar{A}_d = \begin{bmatrix} A_d \\ 0 \end{bmatrix}, \quad \tilde{E}_2 = E_2 \tag{8}$$

we have the filtering error dynamics as follows:

$$\begin{cases} \eta_{k+1} = \tilde{A}\eta_k + (\gamma_k - \beta)\tilde{A}_1 \eta_k + \tilde{A}_d Z \eta_{k-d} + \tilde{B} w_k \\ \tilde{z}_k := z_k - \hat{z}_k = \tilde{C}\eta_k + C_d Z \eta_{k-d} + D w_k \end{cases} \tag{9}$$

where

$$\begin{aligned} \tilde{A} &= \begin{bmatrix} A + \Delta A & 0 \\ \beta K C_2 & G - \beta K C_2 \end{bmatrix} \\ &= \begin{bmatrix} A & 0 \\ \beta K C_2 & G - \beta K C_2 \end{bmatrix} \\ &\quad + \begin{bmatrix} H \\ 0 \end{bmatrix} F_k [E_1 \quad 0] := \bar{A} + \tilde{H} F_k \tilde{E}_1, \end{aligned} \tag{10}$$

$$\begin{aligned} \tilde{A}_1 &= \begin{bmatrix} 0 & 0 \\ K C_2 & 0 \end{bmatrix}, \quad \tilde{A}_d = \begin{bmatrix} A_d + \Delta A_d \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} A_d \\ 0 \end{bmatrix} + \begin{bmatrix} H \\ 0 \end{bmatrix} F_k E_2 := \bar{A}_d + \tilde{H} F_k \tilde{E}_2, \end{aligned} \tag{11}$$

$$\tilde{B} = \begin{bmatrix} B \\ K D_2 \end{bmatrix}, \quad \tilde{C} = [C \quad -L], \quad Z = [I \quad 0]. \tag{12}$$

Since the filtering error system (9) contains the stochastic quantity (i.e., γ_k), we need to introduce the notion of stochastic stability in the mean-square sense for the filtering error system.

Definition 1: The filtering error system (9) is said to be *exponentially mean-square stable* if, with $w_k = 0$, there exist constants $\alpha > 0$ and $\tau \in (0,1)$ such that

$$\mathbb{E}\{\|\eta_k\|^2\} \leq \alpha\tau^k \sup_{-d\leq i\leq 0} \mathbb{E}\{\|\eta_i\|^2\}, \quad k \in \mathbb{I}^+. \tag{13}$$

Assumption 1: System (1) is exponentially mean-square stable for the whole uncertain domain (2).

Similar to [9], Assumption 1 is made based on the fact that there is no control in the system model (1); therefore, the original system (1) to be estimated has to be exponentially mean-square stable for the whole uncertain domain (2), which is a prerequisite for the filtering error system (9) to be exponentially mean-square stable.

With Definition 1, our objective is to design the filter (7) for the system (1) such that, for all possible missing measurements in (4), the filtering error system is exponentially mean-square stable, and H_∞ robustness performance constraint is satisfied. More specifically, we aim to design a filter such that the filtering error system satisfies the following requirements (Q1) and (Q2), simultaneously:

Q1) the filtering error system (9) is exponentially mean-square stable;

Q2) under the zero-initial condition, the filtering error $\tilde{z}_k$ satisfies

$$\sum_{k=0}^{\infty}\mathbb{E}\{\|\tilde{z}_k\|^2\} < \gamma^2 \sum_{k=0}^{\infty}\mathbb{E}\{\|w_k\|^2\} \tag{14}$$

for all nonzero w_k, where $\gamma > 0$ is a prescribed scalar.

The design problem stated above will be referred to as the robust H_∞ filtering problem with missing measurement.

III. H_∞ Filtering Performance Analysis

In this section, we will provide an H_∞ performance analysis result for the filtering error system (9), which will be used for the filter design in the next section. Before proceeding, we introduce the following useful lemma.

Lemma 1: Let $\Theta_k = [\eta_k^T, \eta_{k-1}^T, \ldots, \eta_{k-d}^T]^T$ where η_k is defined in (8). Consider a Lyapunov functional

$$V_k(\Theta_k) = \eta_k^T P \eta_k + \sum_{i=k-d}^{k-1} \eta_i^T Z^T Q Z \eta_i \tag{15}$$

where P and Q are positive definite matrices, and Z is defined in (12). If there exists a scalar $\psi > 0$ such that

$$\mathbb{E}\{V_{k+1}(\Theta_{k+1}) \,|\, \Theta_k\} - V_k(\Theta_k) < -\psi\|\eta_k\|^2 \tag{16}$$

then the dynamics of the process η_k is exponentially mean-square stable.

Proof: It follows readily from (15) that

$$V_k(\Theta_k) \leq \lambda_{\max}(P)\|\eta_k\|^2 + \lambda_{\max}(Z^TQZ) \sum_{i=k-d}^{k-1} \|\eta_i\|^2 \tag{17}$$

which, together with (16), shows that for any scalar $\mu > 1$

$$\begin{aligned}
&\mathbb{E}\{\mu^{k+1}V_{k+1}(\Theta_{k+1})\} - \mathbb{E}\{\mu^k V_k(\Theta_k)\} \\
&\quad = \mu^{k+1}\left[\mathbb{E}\{V_{k+1}(\Theta_{k+1})\} - \mathbb{E}\{V_k(\Theta_k)\}\right] \\
&\qquad + \mu^k(\mu-1)\mathbb{E}\{V_k(\Theta_k)\} \\
&\quad \leq \mu^k[-\mu\psi + (\mu-1)\lambda_{\max}(P)]\mathbb{E}\{\|\eta_k\|^2\} \\
&\qquad + \mu^k(\mu-1)\lambda_{\max}(Z^TQZ)\sum_{i=k-d}^{k-1}\mathbb{E}\{\|\eta_i\|^2\}.
\end{aligned} \tag{18}$$

For any integer $N \geq d+1$, summing up both sides of (17) from 0 to N with respect to k, we have

$$\begin{aligned}
&\mathbb{E}\left\{\mu^N V_N(\Theta_N)\right\} - \mathbb{E}\{V_0(\Theta_0)\} \\
&\quad \leq a(\mu)\sum_{k=0}^{N-1}\mu^k\mathbb{E}\left\{\|\eta_k\|^2\right\} \\
&\qquad + b(\mu)\sum_{k=0}^{N-1}\sum_{i=k-d}^{k-1}\mu^k\mathbb{E}\left\{\|\eta_i\|^2\right\}
\end{aligned} \tag{19}$$

where

$$\begin{aligned}
a(\mu) &= -\mu\psi + (\mu-1)\lambda_{\max}(P) \\
b(\mu) &= (\mu-1)\lambda_{\max}(Z^TQZ).
\end{aligned} \tag{20}$$

Note that for $d \geq 1$

$$\begin{aligned}
&\sum_{k=0}^{N-1}\sum_{i=k-d}^{k-1}\mu^k\mathbb{E}\left\{\|\eta_i\|^2\right\} \\
&\leq \left(\sum_{i=-d}^{-1}\sum_{k=0}^{i+d} + \sum_{i=0}^{N-d-1}\sum_{k=i+1}^{i+d} + \sum_{i=N-d}^{N-1}\sum_{k=i+1}^{N-1}\right)\mu^k\mathbb{E}\left\{\|\eta_i\|^2\right\} \\
&\leq \frac{\mu^d-1}{\mu-1}\sum_{i=-d}^{-1}\mathbb{E}\left\{\|\eta_i\|^2\right\} + \frac{\mu(\mu^d-1)}{\mu-1}\sum_{i=0}^{N-1}\mu^i\mathbb{E}\left\{\|\eta_i\|^2\right\} \\
&\quad + \frac{\mu(\mu^{d-1}-1)}{\mu-1}\sum_{i=0}^{N-1}\mu^i\mathbb{E}\left\{\|\eta_i\|^2\right\}.
\end{aligned} \tag{21}$$

Then, it follows from (19) and (21) that

$$\begin{aligned}
&\mathbb{E}\left\{\mu^N V_N(\Theta_N)\right\} - \mathbb{E}\{V_0(\Theta_0)\} \\
&\quad \leq \frac{b(\mu)(\mu^d-1)d}{\mu-1}\sup_{-d\leq i\leq 0}\mathbb{E}\left\{\|\eta_i\|^2\right\} \\
&\qquad + \zeta(\mu)\sum_{k=0}^{N-1}\mu^k\mathbb{E}\left\{\|\eta_k\|^2\right\}
\end{aligned} \tag{22}$$

where

$$\zeta(\mu) = a(\mu) + \frac{2\mu b(\mu)(\mu^d-1)}{\mu-1}.$$

Since $\zeta(1) = -\psi < 0$ and $\lim_{\mu\to+\infty}\zeta(\mu) = +\infty$, there exists a scalar $\mu_0 > 1$ such that $\zeta(\mu_0) = 0$. Therefore, we can obtain from (22) that, for any integer $N \geq d+1$

$$\begin{aligned}
&\mathbb{E}\left\{\mu_0^N V_N(\Theta_N)\right\} - \mathbb{E}\{V_0(\Theta_0)\} \\
&\quad \leq d\lambda_{\max}(Z^TQZ)\left(\mu_0^d - 1\right)\sup_{-d\leq i\leq 0}\mathbb{E}\left\{\|\eta_i\|^2\right\}.
\end{aligned} \tag{23}$$

Since

$$\mathbb{E}\left\{\mu_0^N V_N(\Theta_N)\right\} \geq \lambda_{\min}(P)\|\eta_N\|^2 \tag{24}$$

and

$$\mathbb{E}\{V_0(\Theta_0)\} \leq d\max(\lambda_{\max}(P), \lambda_{\max}(Z^TQZ)) \sup_{-d\leq i\leq 0} \mathbb{E}\left\{\|\eta_i\|^2\right\} \tag{25}$$

therefore, it follows from (23) that (26), shown at the bottom of the page, holds. Finally, it follows easily from Definition 1 that the dynamics of the process η_k is exponentially mean-square stable. The proof is complete. ■

The following theorem provides sufficient conditions under which the filtering error system (9) is exponentially stable in the mean-square sense and the filtering error $\tilde{z}_k$ satisfies the H_∞ disturbance attenuation level given in (14).

Theorem 1: Given a scalar $\gamma > 0$ and the filter parameters G, K, and L. If there exist positive-definite matrices $P = P^T > 0$ and $Q = Q^T > 0$ satisfying (27), shown at the bottom of the page, where

$$\Sigma = \tilde{A}^TP\tilde{A} + Z^TQZ - P + (1-\beta)\beta\tilde{A}_1^TP\tilde{A}_1 + \tilde{C}^T\tilde{C}, \tag{28}$$

then the filtering error system (9) is exponentially mean-square stable and the filtering error $\tilde{z}_k$ satisfies (14).

Proof: Define the Lyapunov functional in (15) and calculate the difference of the Lyapunov functional from (9) with $w_k = 0$, as follows:

$$\begin{aligned}\Delta V_k &:= \mathbb{E}\left\{V_{k+1}(\Theta_{k+1})|\Theta_k\right\} - V_k(\Theta_k)\\ &= \mathbb{E}\left\{\eta_{k+1}^TP\eta_{k+1}\right\} + \eta_k^T(Z^TQZ-P)\eta_k\\ &\quad - \eta_{k-d}^TZ^TQZ\eta_{k-d}\\ &= \eta_k^T\tilde{A}^TP\tilde{A}\eta_k + \eta_k^T\tilde{A}^TP\tilde{A}_dZ\eta_{k-d}\\ &\quad + \eta_{k-d}^TZ^T\tilde{A}_d^TP\tilde{A}\eta_k + \mathbb{E}\left\{(\gamma_k-\beta)^2\right\}\eta_k^T\tilde{A}_1^TP\tilde{A}_1\eta_k\\ &\quad + \eta_{k-d}^TZ^T\tilde{A}_d^TP\tilde{A}_dZ\eta_{k-d} + \eta_k^T(Z^TQZ-P)\eta_k\\ &\quad - \eta_{k-d}^TZ^TQZ\eta_{k-d}.\end{aligned} \tag{29}$$

Noting that

$$\mathbb{E}\left\{(\gamma_k-\beta)^2\right\} = (1-\beta)\beta \tag{30}$$

we have (31), shown at the bottom of the page. It follows from (27) that there exists a positive scalar $\psi > 0$ such that

$$\begin{bmatrix}\tilde{A}^TP\tilde{A} + Z^TQZ - P + (1-\beta)\beta\tilde{A}_1^TP\tilde{A}_1 & \tilde{A}^TP\tilde{A}_d\\ \tilde{A}_d^TP\tilde{A} & \tilde{A}_d^TP\tilde{A}_d - Q\end{bmatrix} < \begin{bmatrix}-\psi I & 0\\ 0 & 0\end{bmatrix} \tag{32}$$

and subsequently

$$\mathbb{E}\{V_{k+1}(\Theta_{k+1})|\Theta_k\} - V_k(\Theta_k) < -\psi\eta_k^T\eta_k = -\psi\|\eta_k\|^2. \tag{33}$$

We can verify from Lemma 1 that the filtering error system (9) is exponentially mean-square stable.

Next, for any nonzero w_k, (34), shown at the bottom of the next page, follows from (9) and (31), where Σ is defined in (28).

It can be seen from (27) and (34) that

$$\mathbb{E}\{V_{k+1}(\Theta_{k+1})|\Theta_k\} - \mathbb{E}\{V_k(\Theta_k)\} + \mathbb{E}\left\{\tilde{z}_k^T\tilde{z}_k\right\} - \gamma^2\mathbb{E}\left\{w_k^Tw_k\right\} < 0. \tag{35}$$

Now, summing up (35) from 0 to ∞ with respect to k yields

$$\sum_{k=0}^{\infty}\left[\mathbb{E}\{V_{k+1}(\Theta_{k+1})|\Theta_k\} - \mathbb{E}\{V_k(\Theta_k)\} + \mathbb{E}\left\{\tilde{z}_k^T\tilde{z}_k\right\} - \gamma^2\mathbb{E}\left\{w_k^Tw_k\right\}\right] < 0 \tag{36}$$

i.e.,

$$\sum_{k=0}^{\infty}\mathbb{E}\left\{\|\tilde{z}_k\|^2\right\} < \gamma^2\sum_{k=0}^{\infty}\mathbb{E}\left\{\|w_k\|^2\right\} + \mathbb{E}\{V_0\} - \mathbb{E}\{V_\infty\}. \tag{37}$$

Since the system (9) is exponentially mean-square stable, it is straightforward to see that

$$\sum_{k=0}^{\infty}\mathbb{E}\left\{\|\tilde{z}_k\|^2\right\} < \gamma^2\sum_{k=0}^{\infty}\mathbb{E}\left\{\|w_k\|^2\right\} \tag{38}$$

$$\mathbb{E}\left\{\|\eta_N\|^2\right\} \leq \mu_0^{-N}\frac{\left[d\lambda_{\max}(Z^TQZ)\left(\mu_0^d-1\right) + d\max\left(\lambda_{\max}(P), \lambda_{\max}(Z^TQZ)\right)\right]}{\lambda_{\min}(P)} \sup_{-d\leq i\leq 0}\mathbb{E}\left\{\|\eta_i\|^2\right\}. \tag{26}$$

$$\begin{bmatrix}\Sigma & \tilde{A}^TP\tilde{A}_d + \tilde{C}^TC_d & \tilde{A}^TP\tilde{B} + \tilde{C}^TD\\ \tilde{A}_d^TP\tilde{A} + C_d^T\tilde{C} & \tilde{A}_d^TP\tilde{A}_d - Q + C_d^TC_d & \tilde{A}_d^TP\tilde{B} + C_d^TD\\ \tilde{B}^TP\tilde{A} + D^T\tilde{C} & \tilde{B}^TP\tilde{A}_d + D^TC_d & D^TD + \tilde{B}^TP\tilde{B} - \gamma^2I\end{bmatrix} < 0 \tag{27}$$

$$\Delta V_k = \begin{bmatrix}\eta_k\\ Z\eta_{k-d}\end{bmatrix}^T\begin{bmatrix}\tilde{A}^TP\tilde{A} + Z^TQZ - P + (1-\beta)\beta\tilde{A}_1^TP\tilde{A}_1 & \tilde{A}^TP\tilde{A}_d\\ \tilde{A}_d^TP\tilde{A} & \tilde{A}_d^TP\tilde{A}_d - Q\end{bmatrix}\begin{bmatrix}\eta_k\\ Z\eta_{k-d}\end{bmatrix}. \tag{31}$$

under the zero initial condition. This completes the proof. ■

IV. H_∞ Filter Design

According to H_∞ performance analysis conducted in the preceding section, we are now in the position to provide a solution to the H_∞ filtering problem for stochastic time-delay systems with missing measurements. Before giving our main result, we introduce the following well-known lemmas.

Lemma 2 (S-procedure) [3]: Let $M = M^T, H$ and E be real matrices of appropriate dimensions, with F satisfying (2), then

$$M + HFE + E^T F^T H^T < 0 \tag{39}$$

if and only if there exists a positive scalar $\varepsilon > 0$ such that

$$M + \frac{1}{\varepsilon} HH^T + \varepsilon E^T E < 0, \tag{40}$$

or equivalently

$$\begin{bmatrix} M & H & \varepsilon E^T \\ H^T & -\varepsilon I & 0 \\ \varepsilon E & 0 & -\varepsilon I \end{bmatrix} < 0. \tag{41}$$

In the following theorem, it is shown that the addressed filter design problem is solvable for all admissible parameter uncertainties and missing measurements if an LMI is feasible.

Theorem 2: The system (9) is exponentially mean-square stable and the H_∞-norm constraint (14) is achieved for all nonzero w_k if, for the given scalar $\gamma > 0$, there exist positive definite matrices $R = R^T > 0, S = S^T > 0$, and $Q = Q^T > 0$, real matrices Q_1, Q_2 and Q_3, and a real scalar $\varepsilon > 0$ such that LMI (42), shown at the bottom of the page, holds, where $\alpha_1 = [(1-\beta)\beta]^{1/2}$. Moreover, if (42) is true, the desired filter parameters are given by

$$G = X_{12}^{-1} Q_1 (S-R)^{-1} X_{12} + \beta K C_2 \tag{43}$$

$$K = X_{12}^{-1} Q_2 \tag{44}$$

$$L = Q_3 (S-R)^{-1} X_{12} \tag{45}$$

where the matrix X_{12} comes from the factorization $I - RS^{-1} = X_{12} Y_{12}^T < 0$.

Proof: We now prove that (42) is the sufficient condition for the system (9) to be exponentially mean-square stable and also achieve the H_∞-norm constraint (14). Using the Schur complement [3] to (27) and performing the congruence transformations, we can show that (27) is true if and only if the following inequality holds:

$$\begin{bmatrix} -P & 0 & P\tilde{A} & P\tilde{A}_d & P\tilde{B} & 0 \\ 0 & -I & \tilde{C} & C_d & D & 0 \\ \tilde{A}^T P & \tilde{C}^T & -P + Z^T Q Z & 0 & 0 & \alpha_1 \tilde{A}_1^T P \\ \tilde{A}_d^T P & C_d^T & 0 & -Q & 0 & 0 \\ \tilde{B}^T P & D^T & 0 & 0 & -\gamma^2 I & 0 \\ 0 & 0 & \alpha_1 P \tilde{A}_1 & 0 & 0 & -P \end{bmatrix} < 0. \tag{46}$$

Note that (46) can be written in the form of (39) as

$$M + HFE + E^T F^T H^T < 0 \tag{47}$$

$$\mathbb{E}\{V_{k+1}(\Theta_{k+1})|\Theta_k\} - \mathbb{E}\{V_k(\Theta_k)\} + \mathbb{E}\left\{\tilde{z}_k^T \tilde{z}_k\right\} - \gamma^2 \mathbb{E}\left\{w_k^T w_k\right\}$$
$$= \mathbb{E}\left\{ \begin{bmatrix} \eta_k \\ Z\eta_{k-d} \\ w_k \end{bmatrix}^T \begin{bmatrix} \Sigma & \tilde{A}^T P \tilde{A}_d + \tilde{C}^T C_d & \tilde{A}^T P \tilde{B} + \tilde{C}^T D \\ \tilde{A}_d^T P \tilde{A} + C_d^T \tilde{C} & \tilde{A}_d^T P \tilde{A}_d - Q + C_d^T C_d & \tilde{A}_d^T P \tilde{B} + C_d^T D \\ \tilde{B}^T P \tilde{A} + D^T \tilde{C} & \tilde{B}^T P \tilde{A}_d + D^T C_d & D^T D + \tilde{B}^T P \tilde{B} - \gamma^2 I \end{bmatrix} \begin{bmatrix} \eta_k \\ Z\eta_{k-d} \\ w_k \end{bmatrix} \right\} \tag{34}$$

$$\begin{bmatrix} -S & -S & 0 & SA & SA & SA_d & SB & 0 & 0 & SH & 0 & 0 \\ * & -R & 0 & (RA + \beta Q_2 C_2 + Q_1) & RA + \beta Q_2 C_2 & RA_d & RB + Q_2 D_2 & 0 & 0 & RH & 0 & 0 \\ * & * & -I & (C - Q_3) & C & C_d & D & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -S & -S & 0 & 0 & 0 & \alpha_1 C_2^T Q_2^T & 0 & \varepsilon E_1^T & Q \\ * & * & * & * & -R & 0 & 0 & 0 & \alpha_1 C_2^T Q_2^T & 0 & \varepsilon E_1^T & Q \\ * & * & * & * & * & -Q & 0 & 0 & 0 & 0 & \varepsilon E_2^T & 0 \\ * & * & * & * & * & * & -\gamma^2 I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -S & -S & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -R & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -\varepsilon I & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & -\varepsilon I & 0 \\ * & * & * & * & * & * & * & * & * & * & * & -Q \end{bmatrix} < 0 \tag{42}$$

where

$$M=\begin{bmatrix} -P & 0 & P\bar{A} & P\bar{A}_d & P\tilde{B} & 0 \\ 0 & -I & \tilde{C} & C_d & D & 0 \\ \bar{A}^T P & \tilde{C}^T & -P+Z^TQZ & 0 & 0 & \alpha_1\tilde{A}_1^T P \\ \bar{A}_d^T P & C_d^T & 0 & -Q & 0 & 0 \\ \tilde{B}^T P & D^T & 0 & 0 & -\gamma^2 I & 0 \\ 0 & 0 & \alpha_1 P\tilde{A}_1 & 0 & 0 & -P \end{bmatrix}$$

$$H=[P\tilde{H} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T$$

$$E=[0 \quad 0 \quad \tilde{E}_1 \quad \tilde{E}_2 \quad 0 \quad 0].$$

By applying Lemma 2 to (47), we know that (47) holds if and only if there exists a positive scalar parameter ε such that the LMI (48), shown at the bottom of the page, holds.

By Schur complement [3], (48) is equivalent to (49), shown at the bottom of the page.

Recall that our goal is to derive the expression of the filter parameters from (7). To do this, we partition P and P^{-1} as

$$P=\begin{bmatrix} R & X_{12} \\ X_{12}^T & X_{22} \end{bmatrix}, \quad P^{-1}=\begin{bmatrix} S^{-1} & Y_{12} \\ Y_{12}^T & Y_{22} \end{bmatrix} \tag{50}$$

where the partitioning of P and P^{-1} is compatible with that of $\bar{A}$ defined in (10), i.e., $R \in R^{n\times n}, X_{12} \in R^{n\times n}, X_{22} \in R^{n\times n}, S \in R^{n\times n}, Y_{12} \in R^{n\times n}, Y_{22} \in R^{n\times n}$. Define

$$T_1=\begin{bmatrix} S^{-1} & I \\ Y_{12}^T & 0 \end{bmatrix}, \quad T_2=\begin{bmatrix} I & R \\ 0 & X_{12}^T \end{bmatrix} \tag{51}$$

which implies that $PT_1=T_2$ and $T_1^T PT_1=T_1^T T_2$.

Now, define the change of filter parameters as follows:

$$\begin{aligned} Q_1 &= X_{12}(G-\beta KC_2)Y_{12}^T S \\ Q_2 &= X_{12}K, \quad Q_3 = LY_{12}^T S. \end{aligned} \tag{52}$$

By applying the congruence transformations $\mathrm{diag}\{T_1, I, T_1, I, I, T_1, I, I, I\}$ to (49), we obtain (53), shown at the bottom of the next page. Once again, performing the congruence transformation $\mathrm{diag}\{S, I, I, S, I, I, I, S, I, I, I, Q\}$ leads to (53) which, by Theorem 1, is a sufficient condition for guaranteeing that the system (9) is exponentially mean-square stable and the H_∞-norm constraint (14) is achieved.

Furthermore, if the LMI (42) is feasible, we have

$$\begin{bmatrix} -S & -S \\ -S & -R \end{bmatrix} < 0$$

or equivalently

$$\begin{bmatrix} S^{-1} & I \\ I & R \end{bmatrix} > 0.$$

It follows directly from $XX^{-1}=I$ that $I-RS^{-1}=X_{12}Y_{12}^T < 0$. Hence, one can always find square and nonsingular X_{12} and Y_{12} [20]. Therefore, (43)–(45) are obtained from (52), which concludes the proof. ∎

Remark 2: The addressed robust H_∞ filter can be obtained by solving the LMI (42) in Theorem 2. Note that the feasibility of LMI can be checked efficiently via interior point method [3]. It is interesting to discuss the extreme cases of $\beta=0$ and $\beta=1$. In case of $\beta=0$, the true measurements are missing at Probability 1, and the LMI (42) would have no solution if we require a good filtering performance. In the example provided in Section V, when $\beta=0$ and $\gamma \le 20$, there is no solution to (42). On the other hand, in the case where $\beta=1$, the missing probability is zero, and $\alpha_1=0$, the LMI (42) can be simplified to an inequality similar to the one in [24].

$$\begin{bmatrix} -P & 0 & P\bar{A} & P\bar{A}_d & P\tilde{B} & 0 & P\tilde{H} & 0 \\ 0 & -I & \tilde{C} & C_d & D & 0 & 0 & 0 \\ \bar{A}^T P & \tilde{C}^T & -P+Z^TQZ & 0 & 0 & \alpha_1\tilde{A}_1^T P & 0 & \varepsilon\tilde{E}_1^T \\ \bar{A}_d^T P & C_d^T & 0 & -Q & 0 & 0 & 0 & \varepsilon\tilde{E}_2^T \\ \tilde{B}^T P & D^T & 0 & 0 & -\gamma^2 I & 0 & 0 & 0 \\ 0 & 0 & \alpha_1 P\tilde{A}_1 & 0 & 0 & -P & 0 & 0 \\ \tilde{H}^T P & 0 & 0 & 0 & 0 & 0 & -\varepsilon I & 0 \\ 0 & 0 & \varepsilon\tilde{E}_1 & \varepsilon\tilde{E}_2 & 0 & 0 & 0 & -\varepsilon I \end{bmatrix} < 0. \tag{48}$$

$$\begin{bmatrix} -P & 0 & P\bar{A} & P\bar{A}_d & P\tilde{B} & 0 & P\tilde{H} & 0 & 0 \\ 0 & -I & \tilde{C} & C_d & D & 0 & 0 & 0 & 0 \\ \bar{A}^T P & \tilde{C}^T & -P & 0 & 0 & \alpha_1\tilde{A}_1^T P & 0 & \varepsilon\tilde{E}_1^T & Z^T \\ \bar{A}_d^T P & C_d^T & 0 & -Q & 0 & 0 & 0 & \varepsilon\tilde{E}_2^T & 0 \\ \tilde{B}^T P & D^T & 0 & 0 & -\gamma^2 I & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_1 P\tilde{A}_1 & 0 & 0 & -P & 0 & 0 & 0 \\ \tilde{H}^T P & 0 & 0 & 0 & 0 & 0 & -\varepsilon I & 0 & 0 \\ 0 & 0 & \varepsilon\tilde{E}_1 & \varepsilon\tilde{E}_2 & 0 & 0 & 0 & -\varepsilon I & 0 \\ 0 & 0 & Z & 0 & 0 & 0 & 0 & 0 & -Q^{-1} \end{bmatrix} < 0. \tag{49}$$

Remark 3: Note that the LMI (42) is a delay-independent sufficient condition. Since the Schur complement and the *S*-procedure do not bring the conservatism, the overall conservatism actually results from the use of the Lyapunov stability theory. In the case that the time-delay is known, a possible way to reduce the conservatism is to define a parameter-dependent Lyapunov functional, and therefore develop delay-dependent conditions (see, e.g., [9], [11], and the references therein).

Up to now, the filter has been designed to satisfy the requirements Q1) and Q2). As a by-product, the results in Theorem 2 also suggest the following optimization problem.

P1): The optimal H_∞ filtering problem for uncertain stochastic time-delay systems with missing measurements is defined by

$$\min_{S>0,R>0,Q>0,Q_1,Q_2,Q_3,\varepsilon} \gamma \quad \text{subject to (42).} \tag{54}$$

On the other hand, in view of (43)–(45), we make the linear transformation on the state estimate

$$\bar{x}_k = X_{12}\hat{x}_k \tag{55}$$

and then obtain a new representation form of the filter as follows:

$$\begin{cases} \bar{x}_{k+1} = \bar{G}\bar{x}_k + \bar{K}y_k \\ \hat{z}_k = \bar{L}\bar{x}_k \end{cases} \tag{56}$$

where

$$\bar{G} = Q_1(S-R)^{-1}, \quad \bar{K} = Q_2, \quad \bar{L} = Q_3(S-R)^{-1}. \tag{57}$$

We can now see from (57) that the filter parameters can be obtained directly by solving LMI (42) without solving $I - RS^{-1} = X_{12}Y_{12}^T$ for X_{12} in (43)–(45).

Remark 4: In many engineering applications, the performances constraints are often specified *a priori*. For example, in Theorem 2, the filter is designed after H_∞ performance is prescribed. However, we could obtain an improved performance by optimizing certain parameters involved in the design. The aim of the problem (P1) is to exploit the design freedom to meet the optimal H_∞ performance.

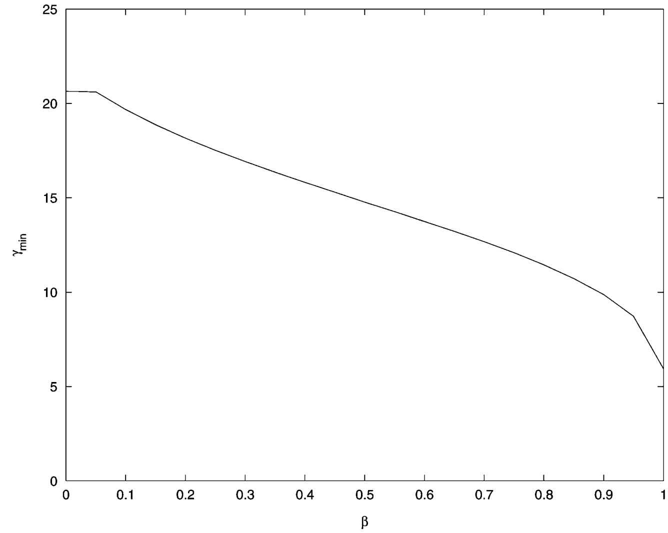

Fig. 1. Probability β versus the optimal H_∞ performance $\gamma_{\min}$.

V. Illustrative Example

In this section, an example is presented to demonstrate the effectiveness and applicability of the proposed method.

Consider the system described by (1) with parameters as follows:

$$A = \begin{bmatrix} -0.9 & 0 & -0.3 \\ 0 & 0.6 & 0.2 \\ 0.5 & 0 & 0.7 \end{bmatrix}, \quad A_d = \begin{bmatrix} -0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.2 \end{bmatrix},$$
$$B = \begin{bmatrix} 0.5 \\ 0 \\ 0.2 \end{bmatrix}, \quad H = \begin{bmatrix} 0.2 \\ 0.3 \\ 0.5 \end{bmatrix},$$
$$E_1 = [0.1 \quad 0 \quad 0], \quad E_2 = [0.1 \quad 0 \quad 0],$$
$$C = [1 \quad 1 \quad 2], \quad C_d = [0.1 \quad 0 \quad 0.5], \quad D = 0.1.$$

The parameters of the output measurements with missing data described by (4) are as follows:

$$C_2 = [1 \quad 2 \quad 1], \quad D_2 = 0.1$$

and $\beta = 0.9$.

$$\begin{bmatrix}
-S^{-1} & -I & 0 & AS^{-1} & A & A_d & B & 0 & 0 & H & 0 & 0 \\
* & -R & 0 & (RA+\beta Q_2C_2+Q_1)S^{-1} & RA+\beta Q_2C_2 & RA_d & RB+Q_2D_2 & 0 & 0 & RH & 0 & 0 \\
* & * & -I & (C-Q_3)S^{-1} & C & C_d & D & 0 & 0 & 0 & 0 & 0 \\
* & * & * & -S^{-1} & -I & 0 & 0 & 0 & \alpha_1 S^{-1}C_2^TQ_2^T & 0 & \varepsilon S^{-1}E_1^T & S^{-1} \\
* & * & * & * & -R & 0 & 0 & 0 & \alpha_1 C_2^TQ_2^T & 0 & \varepsilon E_1^T & I \\
* & * & * & * & * & -Q & 0 & 0 & 0 & 0 & \varepsilon E_2^T & 0 \\
* & * & * & * & * & * & -\gamma^2 I & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -S^{-1} & -I & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & -R & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & -\varepsilon I & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & -\varepsilon I & 0 \\
* & * & * & * & * & * & * & * & * & * & * & -Q^{-1}
\end{bmatrix} < 0. \tag{53}$$

In this case, we would like to provide an optimal H_∞ performance for designing the filter, that is, we are interested in the optimization problem P1). Solving (54) by using the LMI ToolBox, we obtain the minimum value of γ as $\gamma_{\min} = 9.8669$, and

$$R = \begin{bmatrix} 19.4441 & -2.4905 & 7.3716 \\ -2.4905 & 22.8446 & -13.6501 \\ 7.3716 & -13.6501 & 28.9698 \end{bmatrix}$$
$$S = \begin{bmatrix} 12.5872 & -0.1249 & 3.7242 \\ -0.1249 & 2.3690 & -0.4767 \\ 3.7242 & -0.4767 & 8.5188 \end{bmatrix}$$
$$Q = \begin{bmatrix} 1.7339 & -0.2341 & 0.4391 \\ -0.2341 & 1.0196 & -0.7775 \\ 0.4391 & -0.7775 & 3.9319 \end{bmatrix}$$
$$Q_1 = \begin{bmatrix} 5.0702 & 0.8269 & -0.9586 \\ 3.2624 & -12.2390 & 5.0024 \\ -1.7325 & 13.8651 & -9.8236 \end{bmatrix}$$
$$Q_2 = \begin{bmatrix} 0.3171 \\ -0.1183 \\ -3.3411 \end{bmatrix}$$
$$Q_3 = [0.9963 \quad 1.0167 \quad 2.1820], \quad \varepsilon = 20.6692.$$

Hence, the H_∞ robust filter is given by

$$\begin{cases} \bar{x}_{k+1} = \begin{bmatrix} -0.8445 & -0.0186 & 0.1855 \\ -0.3799 & 0.7515 & 0.3072 \\ -0.0048 & -0.6286 & 0.0763 \end{bmatrix} \bar{x}_k + \begin{bmatrix} 0.3171 \\ -0.1183 \\ -3.3411 \end{bmatrix} y_k \\ \hat{z}_k = [-0.0984 \quad -0.2021 \quad -0.2194]\bar{x}_k. \end{cases}$$

In order to understand how the missing measurement affects the H_∞ performance of the filtering process, we now illustrate the interplay between the missing probability and the optimal H_∞ performance. The relationship of β versus $\gamma_{\min}$ is plotted in Fig. 1.

The numerical results in this example show that the filter can be easily obtained by solving an LMI using the Matlab LMI ToolBox, and the optimal H_∞ performance can be obtained by solving the optimal H_∞ filtering problem (54). Furthermore, the results also indicate that the bigger the missing probability, the poorer the H_∞ performance, which is reasonable.

VI. Conclusion

The problem of robust H_∞ filtering has been considered in this paper for stochastic uncertain discrete time-delay systems with missing measurements. The robust H_∞ filter has been designed in terms of a feasible LMI, which guarantee the filtering error system to be exponentially mean-square stable and the filtering error to satisfy H_∞ robustness performance constraint for all possible missing observations and all admissible parameter uncertainties. An optimal filter design problem is also provided by optimizing the H_∞ filtering performances. Our method can be extended to deal with the robust H_∞ control problem.

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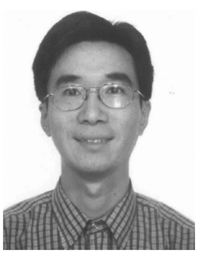

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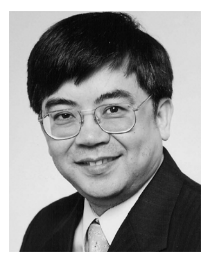

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